Mass segregation in young star clusters

Christoph Olczak

ARI & MPIA, Heidelberg

NAOC, Beijing

Rainer Spurzem, Xiaoying Pang (NAOC, KIAA Beijing; ARI Heidelberg)

Elena Schilbach, Siegfried Röser, Andreas Ernst (ARI Heidelberg)

Thomas Henning, Bertrand Goldmann, Wolfgang Brandner (MPIA Heidelberg)

Andrea Stolte, Maryam Habibi (AIfA Bonn)

Johanna Ziegel (IMSV Bern)

Thordis L. Thorarinsdottir (NR Oslo)

Giacomo Beccari (ESO Garching)

Stefan Harfst (TU Berlin)

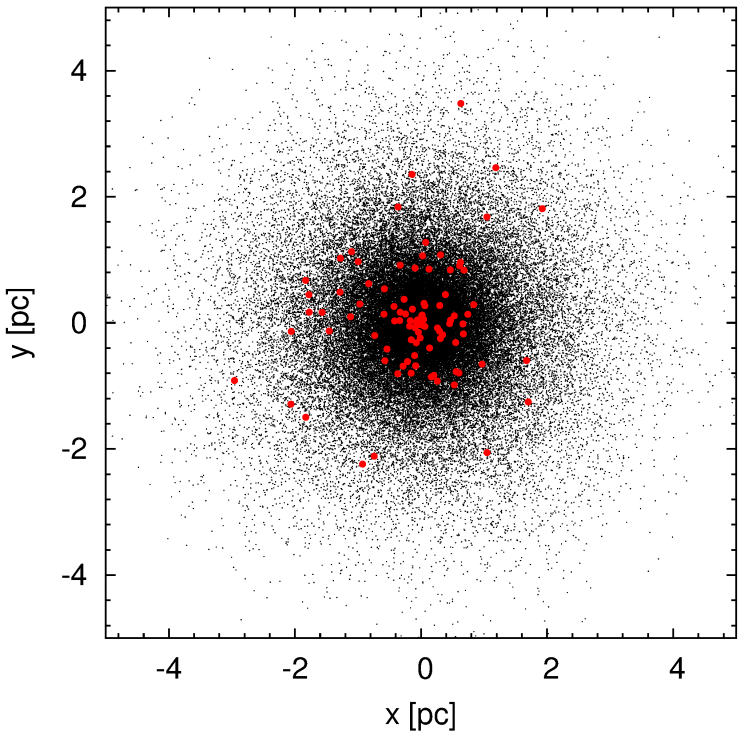
Outline

1. Mass Segregation
   - Introduction
   - Methodology
   - Understanding the evolution of mass segregation
   - Evaluating mass segregation in real star clusters
2. Summary

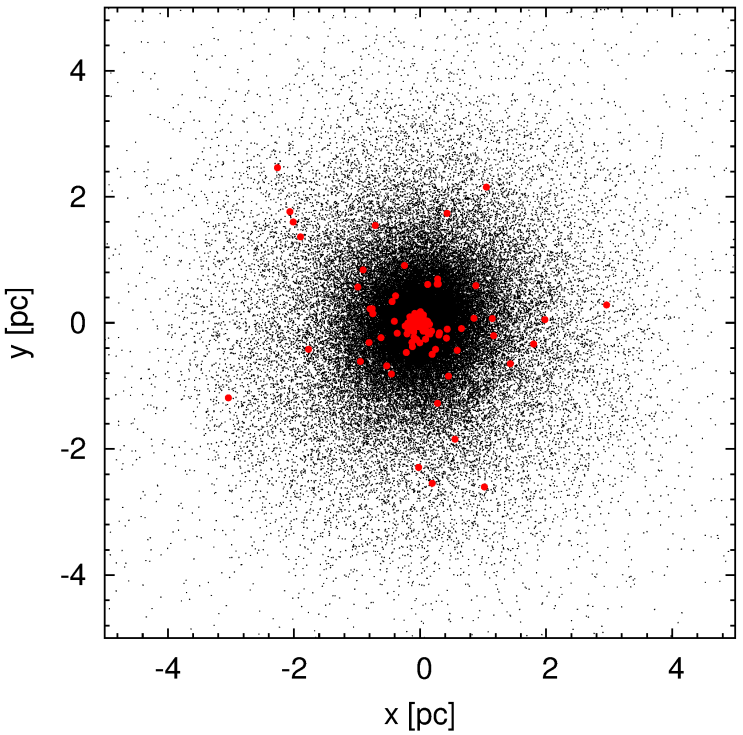
What is mass segregation?

A statistically significant spatial concentration of a sample of stars of a given mass range compared to the entire population.

unsegregated



segregated



Note potential observational bias

Observations usually only provide 2-dimensional spatial information.

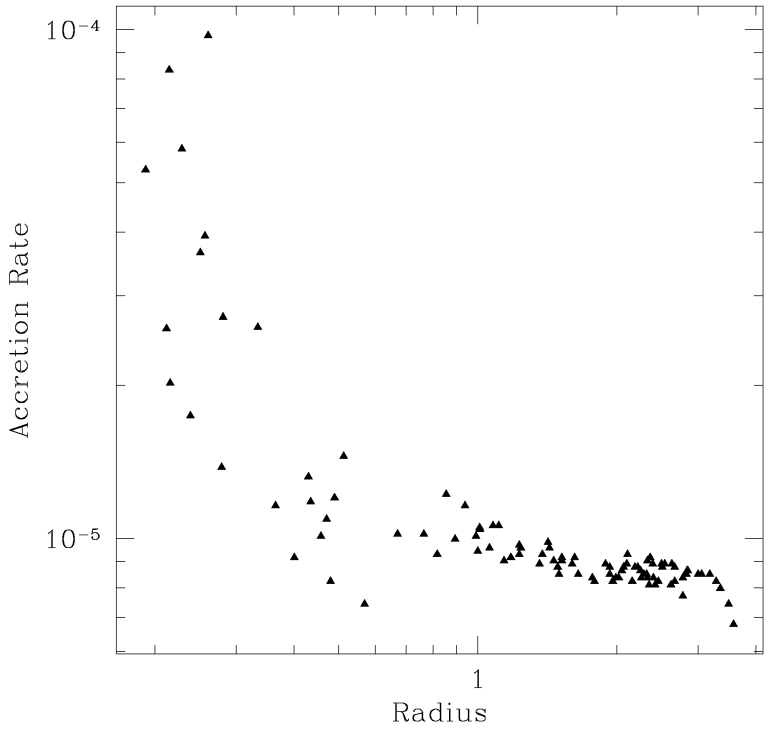
What is the origin of mass segregation?

1. PRIMORDIAL
   - star formation
2. DYNAMICAL
   - two-body relaxation

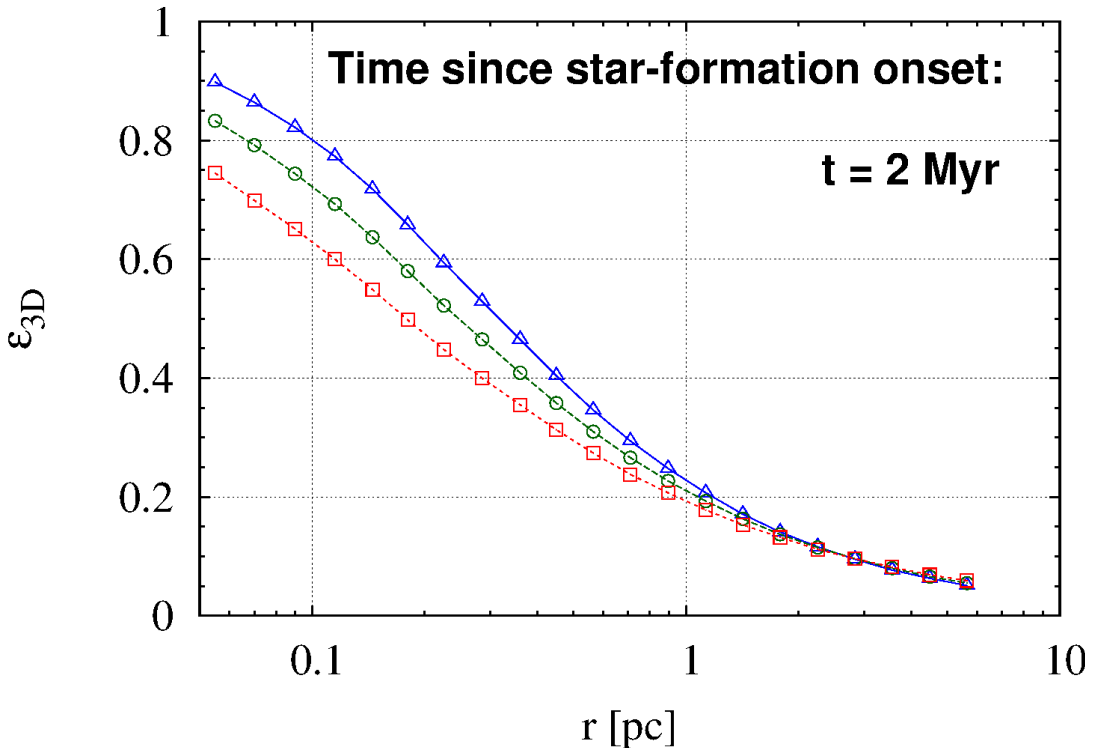
What is the origin of mass segregation?

1. PRIMORDIAL
   - star formation

Bonnell (2000):
"competitive accretion"



Parmentier & Pfalzner (2013):
"local-density driven star formation"



Note: so far there exists no observational evidence for *primordial* mass segregation.

What is the origin of mass segregation?

1. PRIMORDIAL
   - star formation

2. DYNAMICAL
   - two-body relaxation ($\rightarrow$ energy equipartition)

   Relaxation time (Spitzer & Hart, 1971):

$$t_{\rm rel} \;=\; 0.14\,\frac{N}{\ln\left(0.4N\right)}\sqrt{\frac{R_{\rm hm}}{GM}} \;\approx\; 0.1\,\frac{N}{\ln N}\,t_{\rm cross}$$

What is the origin of mass segregation?

1. PRIMORDIAL
   - star formation

2. DYNAMICAL
   - two-body relaxation ($\rightarrow$ energy equipartition)

Relaxation time (Spitzer & Hart, 1971):

$$t_{\rm rel} \;=\; 0.14\,\frac{N}{\ln\left(0.4N\right)}\sqrt{\frac{R_{\rm hm}}{GM}} \;\approx\; 0.1\,\frac{N}{\ln N}\,t_{\rm cross}$$

Spitzer (1969): energy equipartition of self-gravitating systems with two mass groups (where $m_2 \gg m_1, \sum m_2 = M_2 \ll M_{\rm C_1} = \rho_{\rm C_1} r_{\rm C_1}^3$; $\rm C_{\it i}$ denotes the core of group i)

$$t_{\rm eq} \;=\; \frac{\left(\sigma_1^2+\sigma_2^2\right)^{\frac{3}{2}}}{8\sqrt{6\pi}\rho_{\rm C_1} G^2 m_2 \ln N_1} \;=\; \frac{3\sqrt{\pi}}{16}\left(1+\frac{\sigma_2^2}{\sigma_1^2}\right)^{\frac{3}{2}}\frac{m_1}{m_2}\,t_{\rm rel,1} \;\underset{\sigma_1=\sigma_2}{\approx}\; \frac{m_1}{m_2}\,t_{\rm rel,1}$$

What is the origin of mass segregation?

1. PRIMORDIAL
   - star formation

2. DYNAMICAL
   - two-body relaxation ($\rightarrow$ energy equipartition)

Relaxation time (Spitzer & Hart, 1971):

$$t_{\rm rel} \;=\; 0.14\,\frac{N}{\ln\left(0.4N\right)}\sqrt{\frac{R_{\rm hm}}{GM}} \;\approx\; 0.1\,\frac{N}{\ln N}\,t_{\rm cross}$$

Spitzer (1969): energy equipartition of self-gravitating systems with two mass groups (where $m_2 \gg m_1, \sum m_2 = M_2 \ll M_{\rm C_1} = \rho_{\rm C_1} r_{\rm C_1}^3$; $\rm C_{\it i}$ denotes the core of group i)

$$t_{\rm eq} \;=\; \frac{\left(\sigma_1^2+\sigma_2^2\right)^{\frac{3}{2}}}{8\sqrt{6\pi}\rho_{\rm C_1} G^2 m_2 \ln N_1} \;=\; \frac{3\sqrt{\pi}}{16}\left(1+\frac{\sigma_2^2}{\sigma_1^2}\right)^{\frac{3}{2}}\frac{m_1}{m_2}\,t_{\rm rel,1} \;\underset{\sigma_1=\sigma_2}{\approx}\; \frac{m_1}{m_2}\,t_{\rm rel,1}$$

In young massive star clusters ($N \approx 10^4$):

$$\left.\begin{aligned} m_2 &\approx \langle m_{\rm high}\rangle \approx 50\,{\rm M}_\odot \\ m_1 &\approx \langle m_{\rm low}\rangle \;\approx 0.5\,{\rm M}_\odot \end{aligned}\right\} \Rightarrow t_{\rm eq} \approx 10^{-2} t_{\rm rel,1} \approx t_{\rm cross} < 1\,{\rm Myr}\;(!)$$

What is the origin of mass segregation?

1. PRIMORDIAL
   - star formation

2. DYNAMICAL
   - two-body relaxation ($\rightarrow$ energy equipartition)

Mass segregation is a powerful diagnostic tool of self-gravitating stellar systems

- imprint of
  1. formation process ("primordial") *and*
  2. dynamical evolution

- only three diagnostic parameters required
  1. physical age t_*
  2. equipartition time $t_{\rm eq}$

  $\Rightarrow$ degree of dynamical mass segregation $\mu_{\rm dyn} = \mu(t_*, t_{\rm eq})$
  3. apparent degree of mass segregation $\mu_{\rm obs} = \mu(x, y, ...)$

$$\Longrightarrow \begin{cases} \text{primordial mass segregation} & \text{if } \mu_{\rm obs} \gg \mu_{\rm dyn} \\ \text{purely dynamical mass segregation} & \text{if } \mu_{\rm obs} \approx \mu_{\rm dyn} \end{cases}$$

A new efficient measure of mass segregation

Goal

Efficient measure of mass segregation μ for both observational and numerical data:

- geometrically independent,
- independence of quantitative mass measurement,
- numerical robustness, and
- simple, intuitive measure.

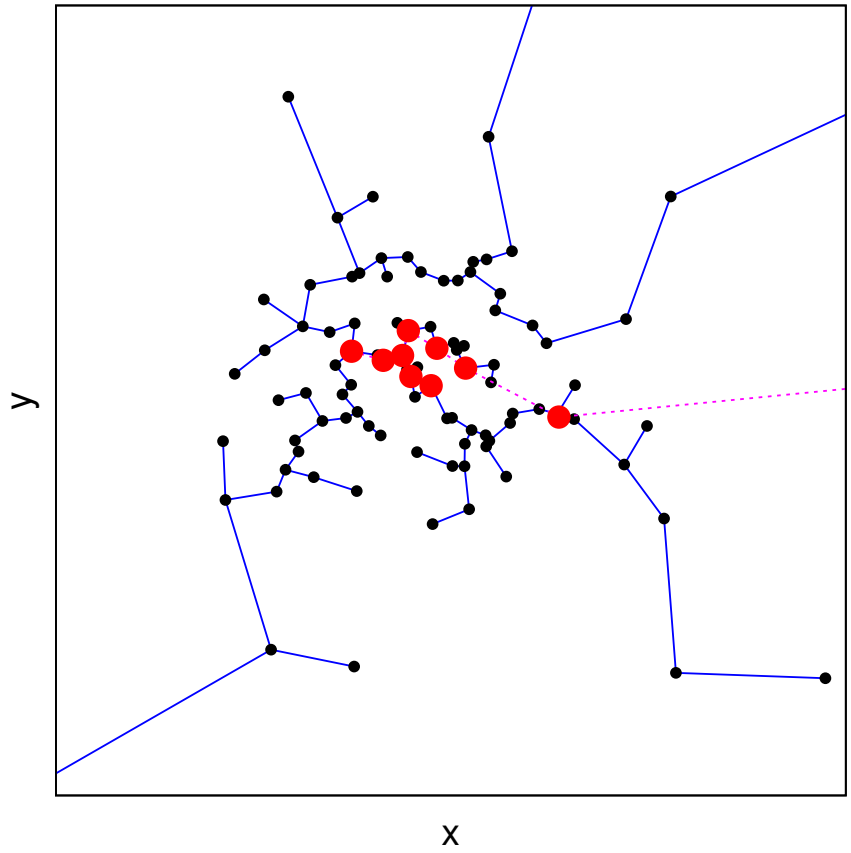
A new efficient measure of mass segregation

Goal

Efficient measure of mass segregation μ for both observational and numerical data:

- geometrically independent,
- independence of quantitative mass measurement,
- numerical robustness, and
- simple, intuitive measure.

$\Rightarrow$ **Minimum Spanning Tree (MST)**



Definition

MST $\equiv$ shortest connecting graph $G = (V, E)$ of all vertices $v_i \in V$ without closed loops,

where
$$V := \{v_1, ..., v_N\} \subset \mathbb{R}^2 \qquad \text{“vertices”}$$
$$E := \{\{v_i, v_j\} \mid v_i, v_j \in V\} \qquad \text{“edges”}$$

Measuring mass segregation via the MST

Construction

1. Construct sub-MST, i.e. shortest connecting subgraph $G' = (V', E')$ of $n < N$ stars, where $V' := \{v'_1, ..., v'_n\} \subset V$, $E' := \{\{v'_i, v'_j\} \mid v'_i, v'_j \in V'\}$.
2. Assign to each edge $e = \{v'_i, v'_j\} \in E'$ the weight $w_e \equiv w_{ij} \equiv ||v'_i - v'_j||$ ("edge length").

Measuring mass segregation via the MST

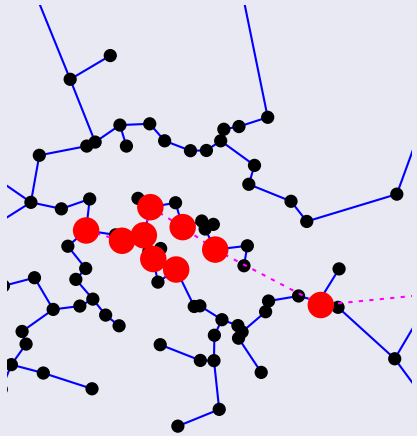
Construction

1. Construct sub-MST, i.e. shortest connecting subgraph $G' = (V', E')$ of $n < N$ stars, where $V' := \{v'_1, ..., v'_n\} \subset V$, $E' := \{\{v'_i, v'_j\} \mid v'_i, v'_j \in V'\}$.
2. Assign to each edge $e = \{v'_i, v'_j\} \in E'$ the weight $w_e \equiv w_{ij} \equiv ||v'_i - v'_j||$ ("edge length").

Quantifying mass segregation

Allison et al. (2009)

1. Define a measure μ of the sub-MST: $\boxed{\lambda = \sum_{e \in E'} w_e}$
2. Calculate μ of the n most massive stars: $\lambda_{\rm MST}^{\rm mass}$
3. Calculate $\bar{\mu}$, $\Delta\mu$ of k sets of n random stars: $\langle \lambda_{\rm MST}^{\rm ref} \rangle$, $\Delta\lambda_{\rm MST}^{\rm ref}$
4. Normalize μ ($\Rightarrow$ **signature** if $\mu > 1$): $\boxed{\Lambda_{\rm MST} = \frac{\langle \lambda_{\rm MST}^{\rm ref} \rangle}{\lambda_{\rm MST}^{\rm mass}}}$
5. Normalize $\Delta\mu$ ($\Rightarrow$ **significance** $\frac{\mu - 1}{\Delta\mu}$): $\boxed{\Delta\Lambda_{\rm MST} = \frac{\Delta\lambda_{\rm MST}^{\rm ref}}{\lambda_{\rm MST}^{\rm mass}}}$



Measuring mass segregation via the MST

Construction

1. Construct sub-MST, i.e. shortest connecting subgraph $G' = (V', E')$ of $n < N$ stars, where $V' := \{v'_1, ..., v'_n\} \subset V$, $E' := \{\{v'_i, v'_j\} \mid v'_i, v'_j \in V'\}$.
2. Assign to each edge $e = \{v'_i, v'_j\} \in E'$ the weight $w_e \equiv w_{ij} \equiv ||v'_i - v'_j||$ ("edge length").

Quantifying mass segregation

	Allison et al. (2009)	Olczak et al. (2011)
1. Define a measure μ of the sub-MST:	$\lambda = \sum_{e \in E'} w_e$	$\gamma = \sqrt[n]{\prod_{e \in E'} w_e}$
2. Calculate μ of the n most massive stars:	$\lambda_{\rm MST}^{\rm mass}$	$\gamma_{\rm MST}^{\rm mass}$
3. Calculate $\bar{\mu}$, $\Delta\mu$ of k sets of n random stars:	$\langle \lambda_{\rm MST}^{\rm ref} \rangle$, $\Delta\lambda_{\rm MST}^{\rm ref}$	$\langle \gamma_{\rm MST}^{\rm ref} \rangle$, $\Delta\gamma_{\rm MST}^{\rm ref}$
4. Normalize μ ($\Rightarrow$ **signature** if $\mu > 1$):	$\Lambda_{\rm MST} = \frac{\langle \lambda_{\rm MST}^{\rm ref} \rangle}{\lambda_{\rm MST}^{\rm mass}}$	$\Gamma_{\rm MST} = \frac{\langle \gamma_{\rm MST}^{\rm ref} \rangle}{\gamma_{\rm MST}^{\rm mass}}$
5. Normalize $\Delta\mu$ ($\Rightarrow$ **significance** $\frac{\mu-1}{\Delta\mu}$):	$\Delta\Lambda_{\rm MST} = \frac{\Delta\lambda_{\rm MST}^{\rm ref}}{\lambda_{\rm MST}^{\rm mass}}$	$\Delta\Gamma_{\rm MST} = \frac{\Delta\gamma_{\rm MST}^{\rm ref}}{\gamma_{\rm MST}^{\rm mass}}$

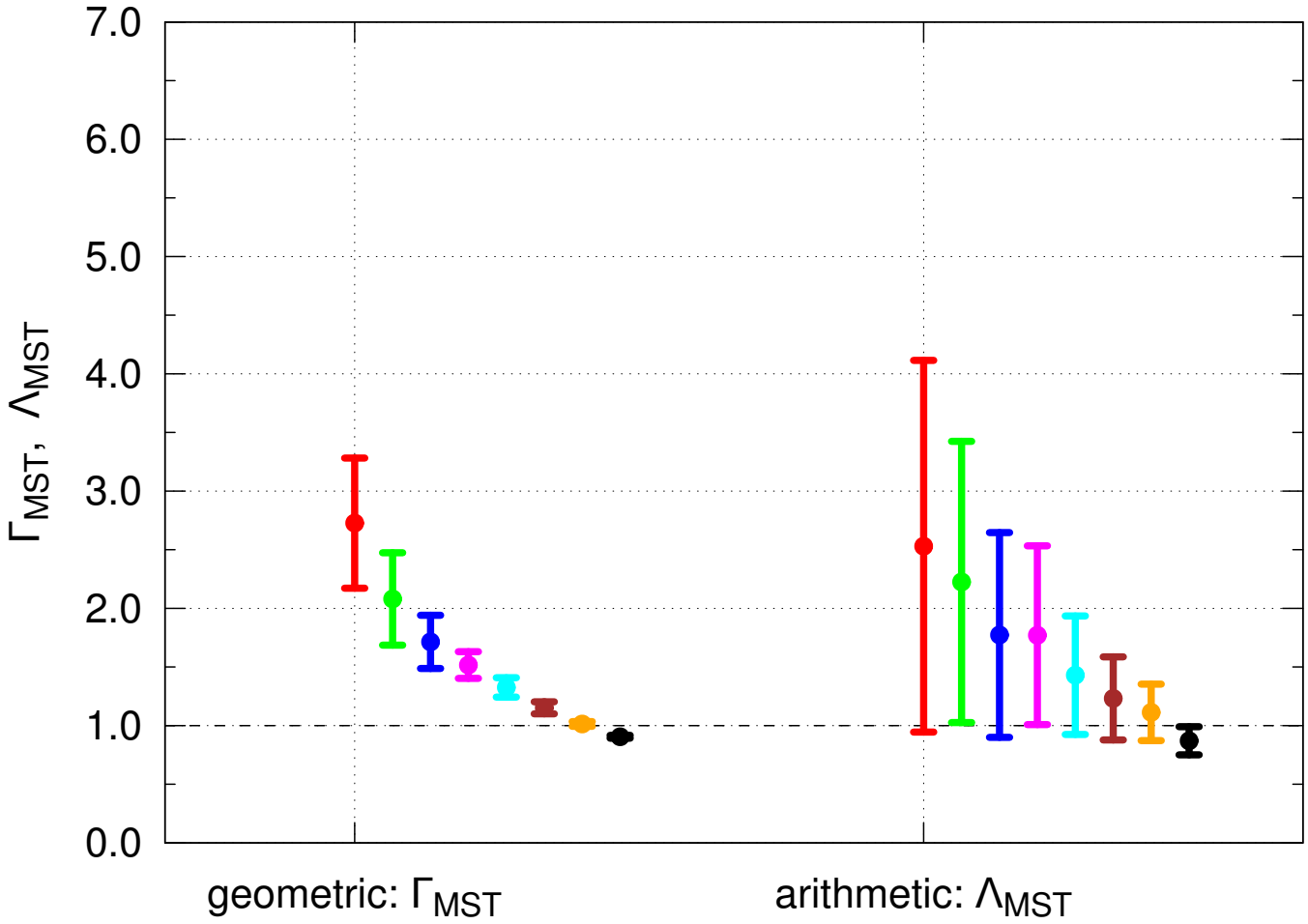
Use the *geometric mean* $\Gamma_{\rm MST}$ of the edges rather than their sum $\Lambda_{\rm MST}$.

$\Rightarrow$ Acts as an intermediate pass that damps contributions from extreme edge lengths.

Comparison of $\Gamma_{\rm MST}$ and $\Lambda_{\rm MST}$ via model star clusters

Star cluster with 10k single stars and Kroupa (2001) mass function in the range $0.08 - 150\,{\rm M}_\odot$.

Initial mass segregation parametrized via $S = 0.3 \in [0, 1)$ following Šubr et al. (2008).



groups of mass ordered stars: 1-50, 51-100, 101-200, 201-500, 501-1000, 1001-2000, 2001-5000, and 5001-10000

The evolution of mass segregation in model star clusters

Fixed parameters

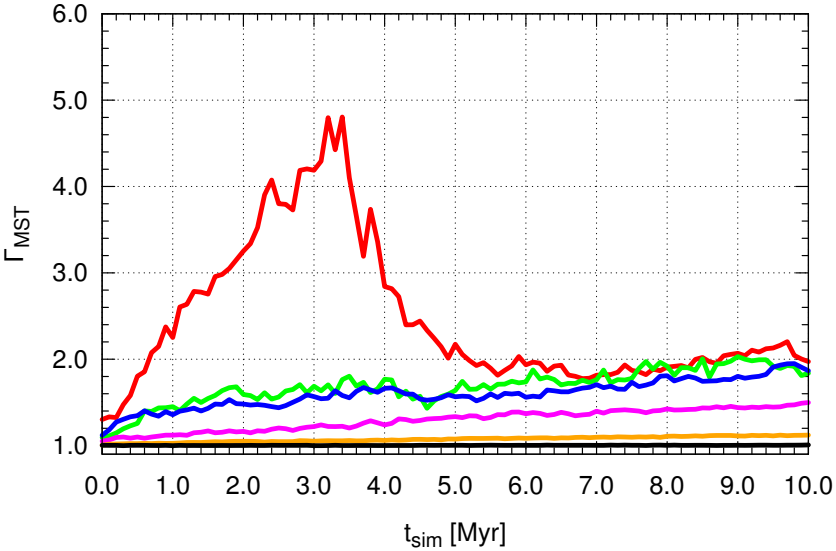
- **no initial mass segregation**
- $N = 10\text{k}$ (centre-of-mass particles)
- $R_{\rm hm} = 1\,\text{pc}$
- IMF: Kroupa (2001) with $m \in [0.08, 150]\,\text{M}_\odot$
- MST groups: 1-**5**, 6-**10**, 11-**20**, 21-**50**, 51-**500**, 501-**5000** most massive stars

Variable parameters

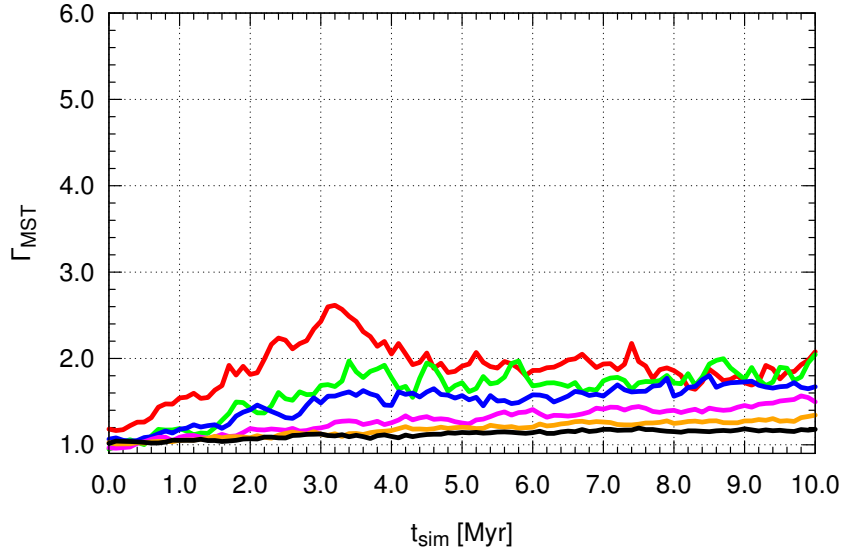
- King model: $W_0 = \{3, 12\}$
- stellar evolution: $SE = \{\text{on}, \text{off}\}$
- virial ratio: $Q = \{0.1, 0.5\}$
- binary fraction: $b_f = \{0.0, 0.1, 1.0\}$

The effect of the density distribution:

$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, SE

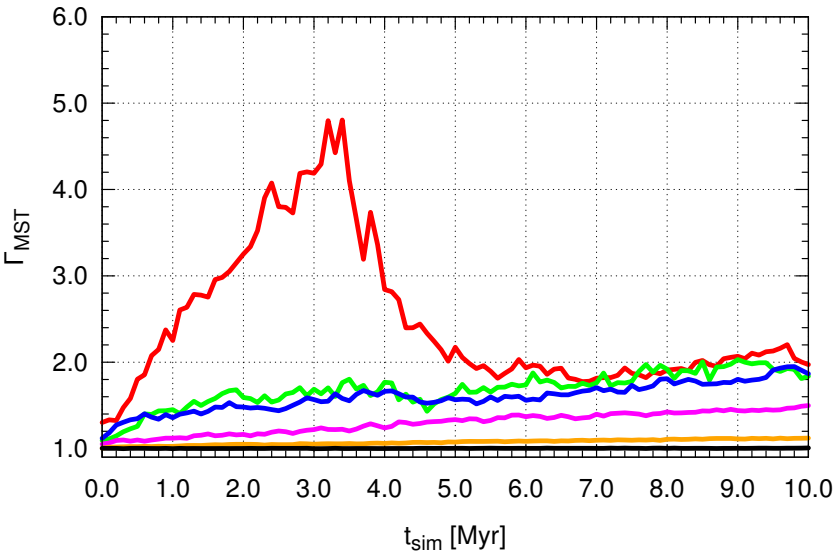


$N = 10k$, $W_0 = 03$, $Q = 0.5$, $b_f = 0.0$, SE

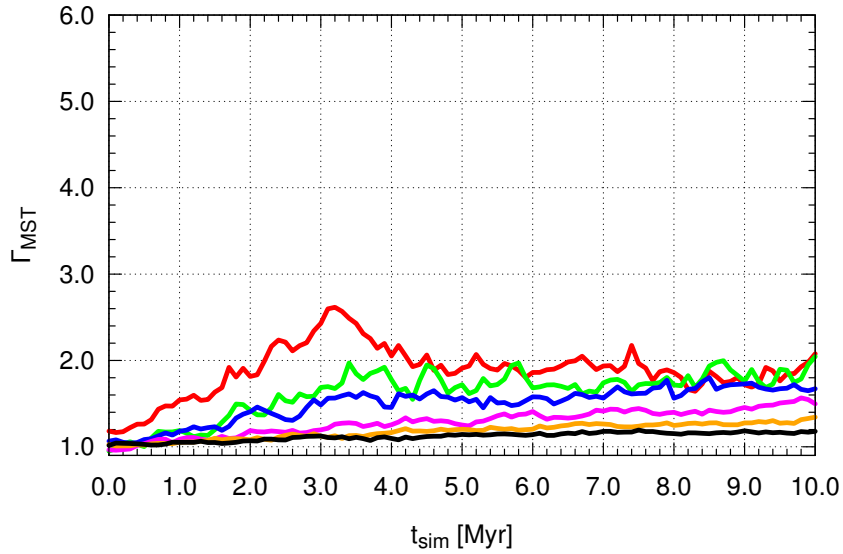


The effect of the density distribution:

$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, *SE*

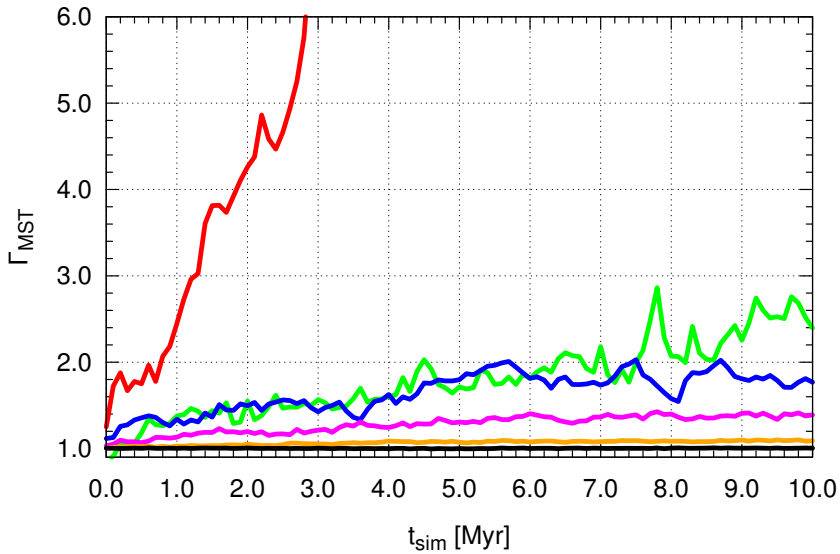


$N = 10k$, $W_0 = 03$, $Q = 0.5$, $b_f = 0.0$, *SE*



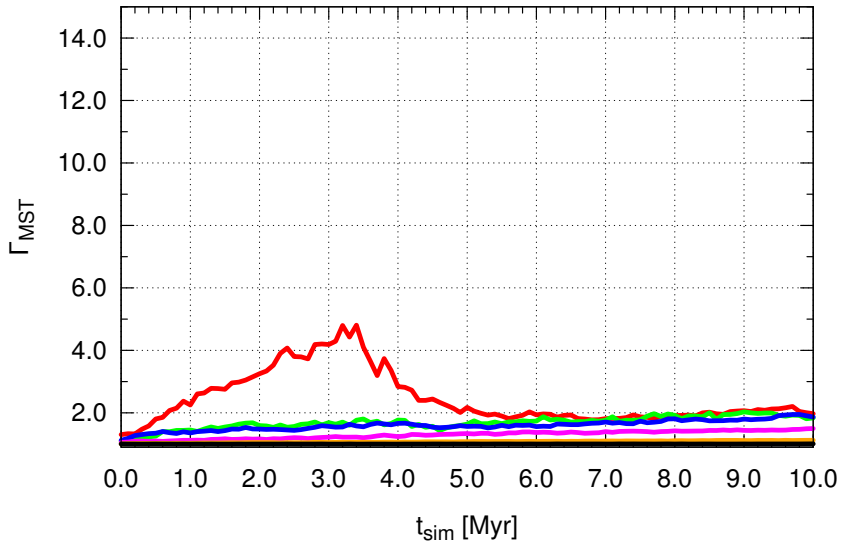
The effect of stellar evolution:

$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, ~~*SE*~~

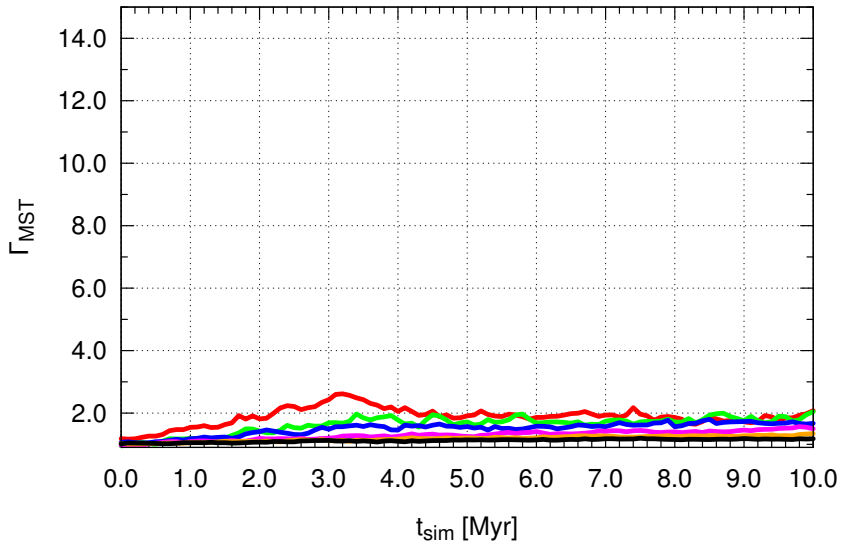


The effect of the density distribution:

$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, *SE*

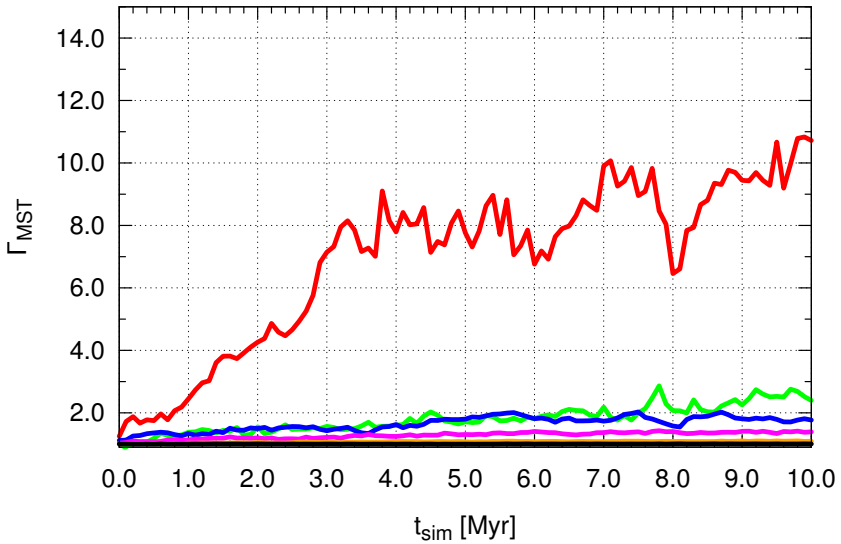


$N = 10k$, $W_0 = 03$, $Q = 0.5$, $b_f = 0.0$, *SE*



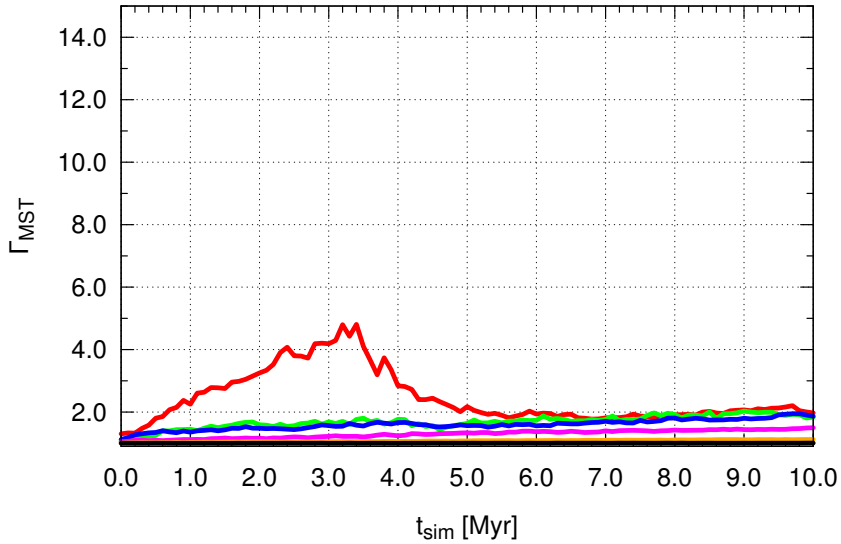
The effect of stellar evolution:

$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, ~~*SE*~~

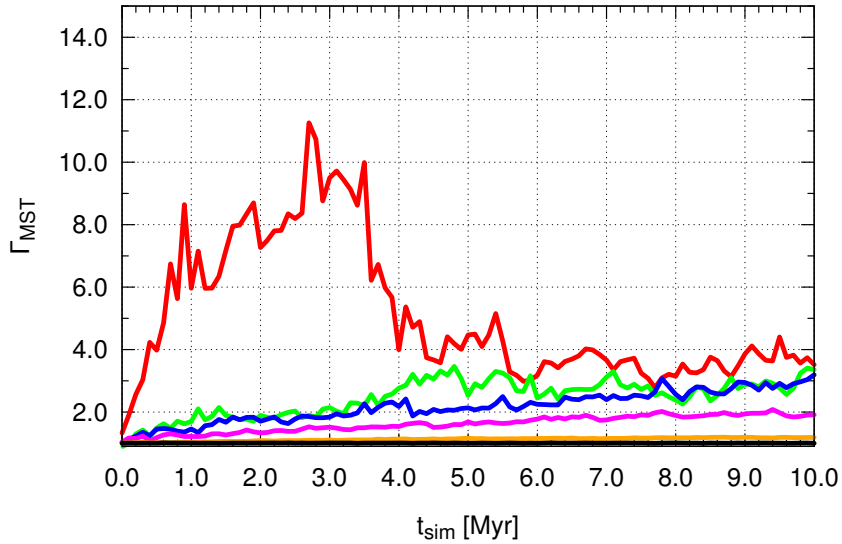


The effect of the initial virial ratio (equilibrium vs. cold initial state):

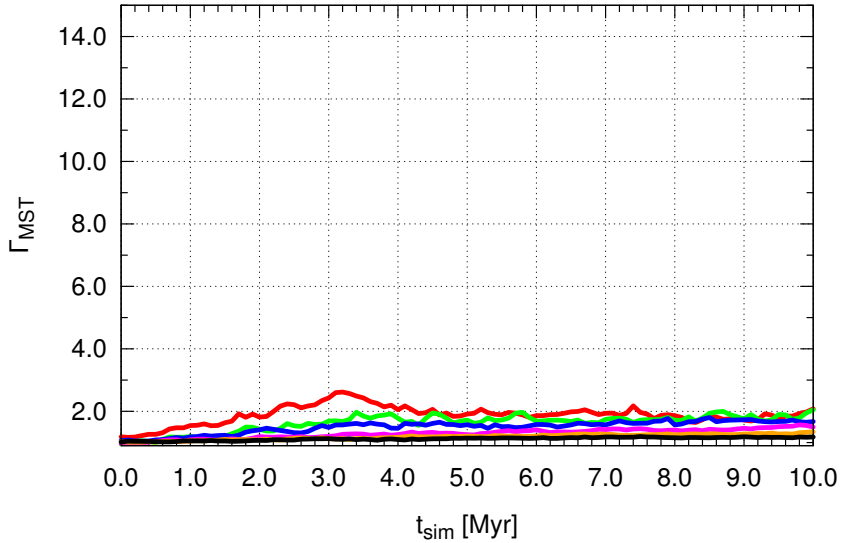
$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, SE



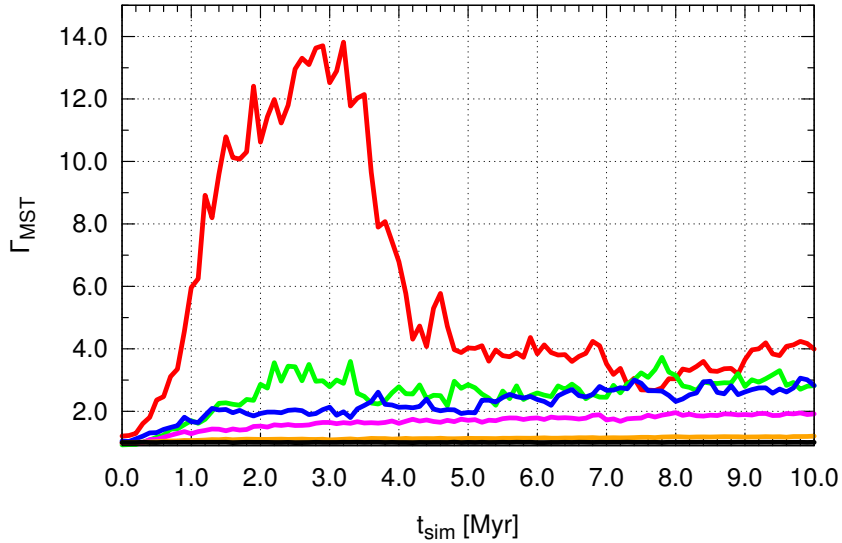
$N = 10k$, $W_0 = 12$, $Q = 0.1$, $b_f = 0.0$, SE



$N = 10k$, $W_0 = 03$, $Q = 0.5$, $b_f = 0.0$, SE

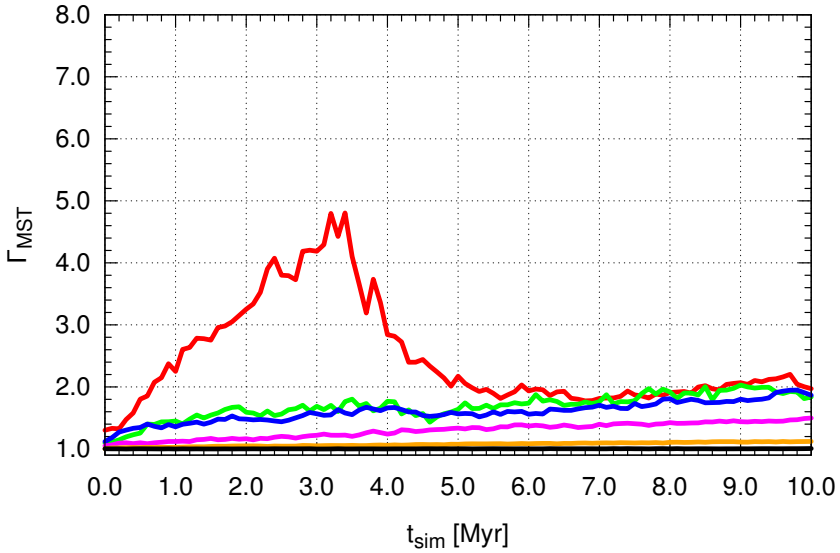


$N = 10k$, $W_0 = 03$, $Q = 0.1$, $b_f = 0.0$, SE

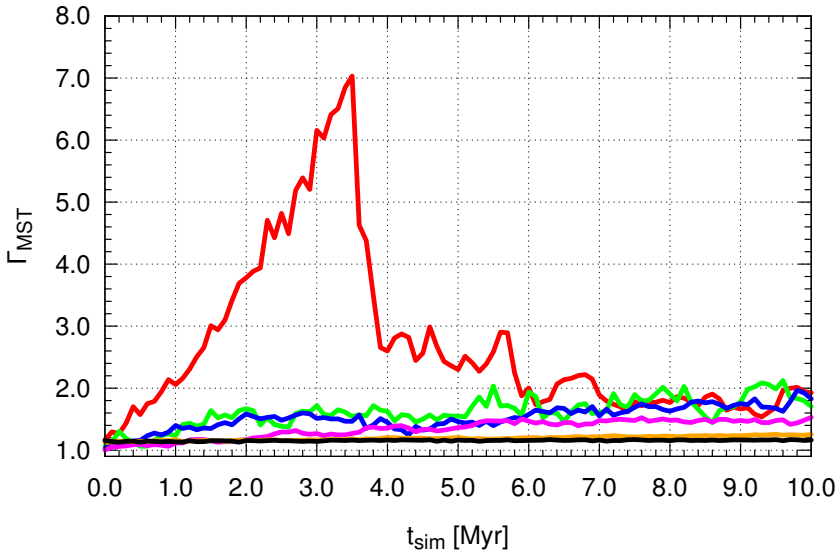


The effect of binaries:

$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, SE

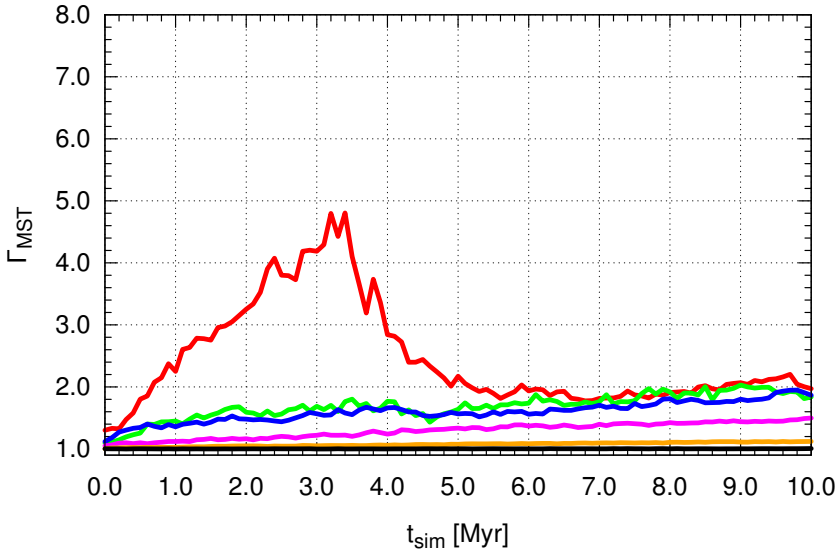


$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.1$, SE

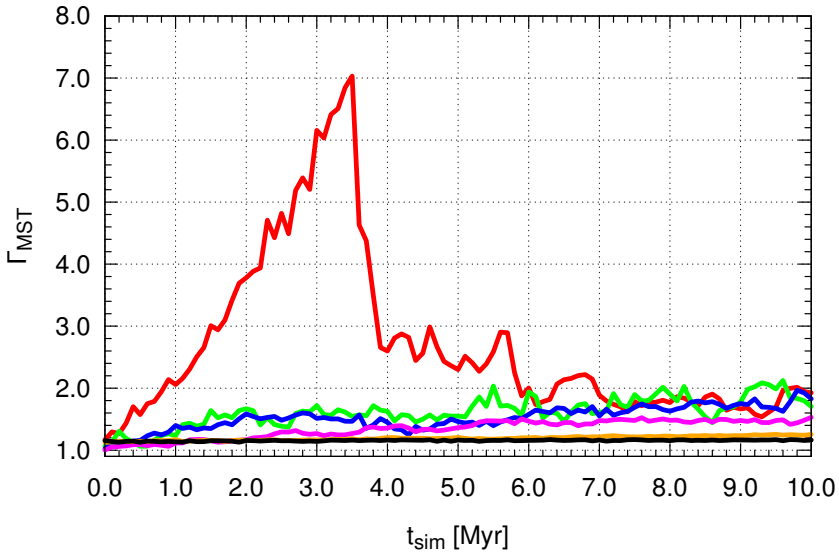


The effect of binaries:

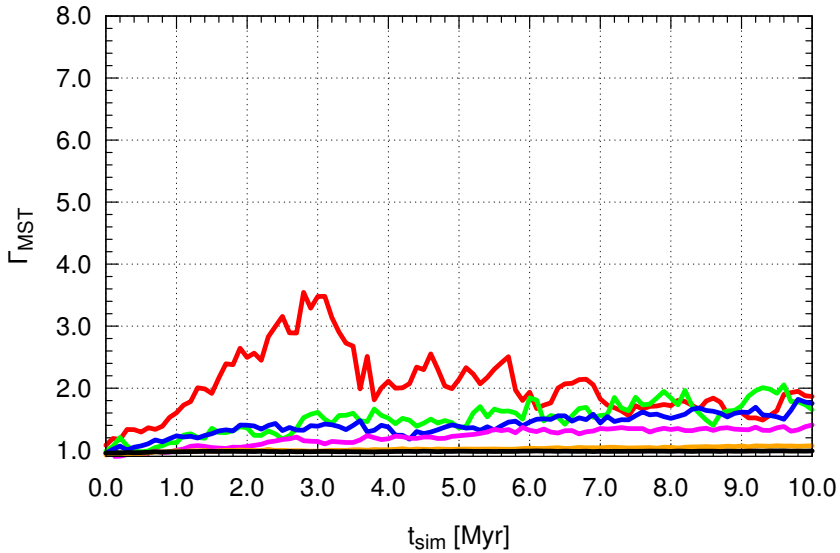
$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, SE



$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.1$, SE

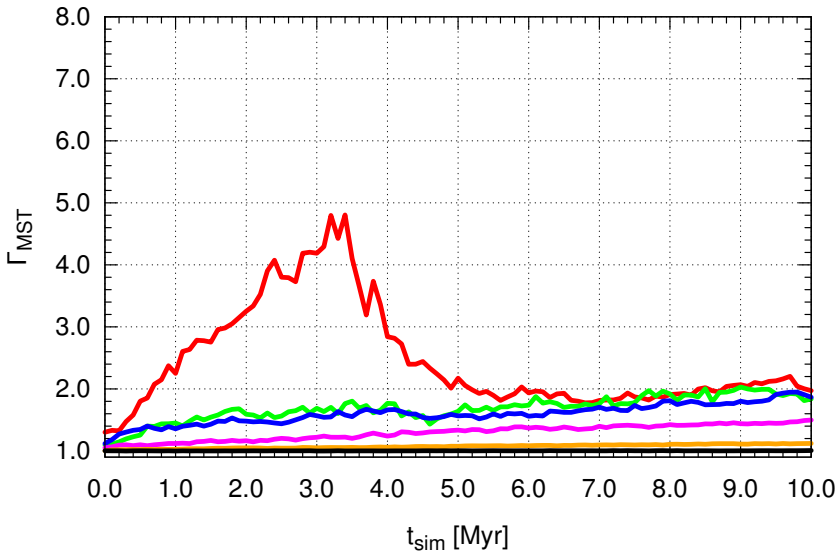


$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.1$, SE, p

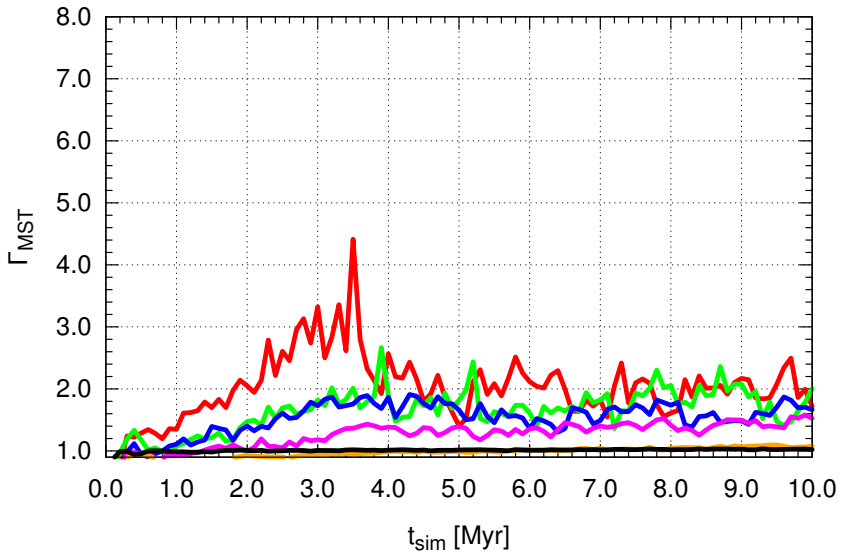


The effect of binaries:

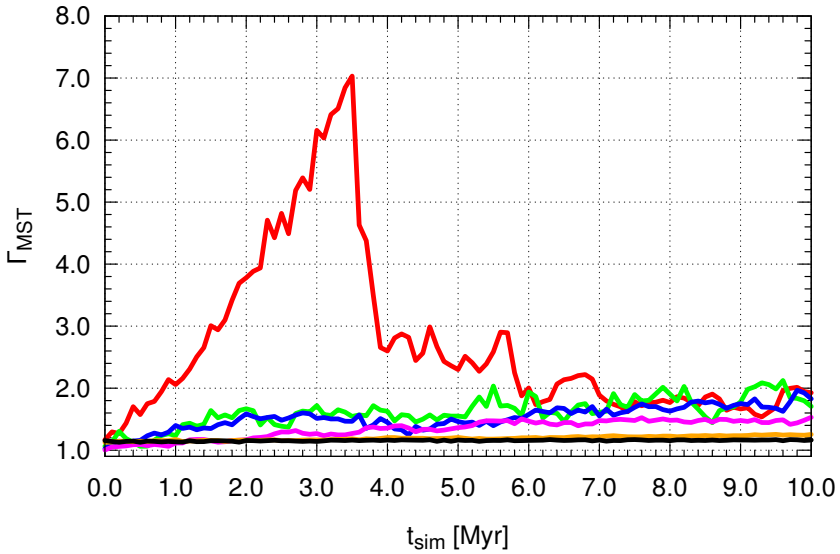
$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.0$, SE



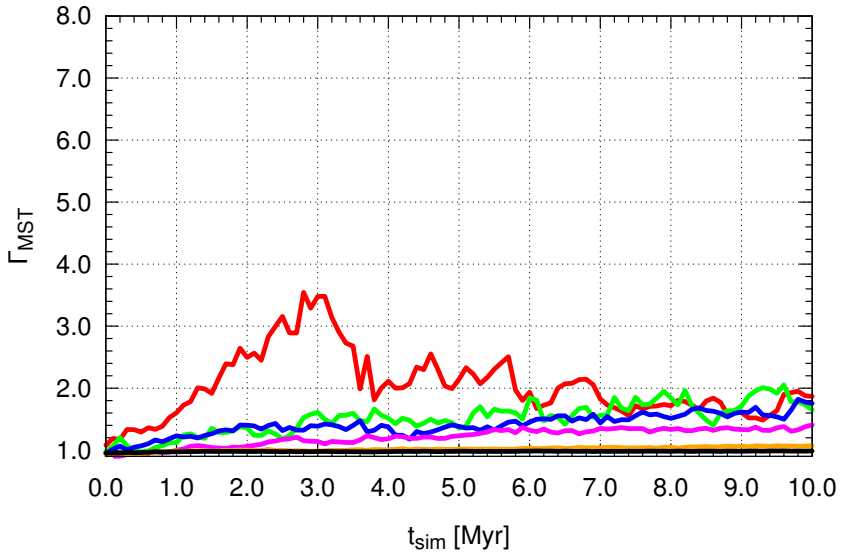
$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 1.0$, SE, p



$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.1$, SE



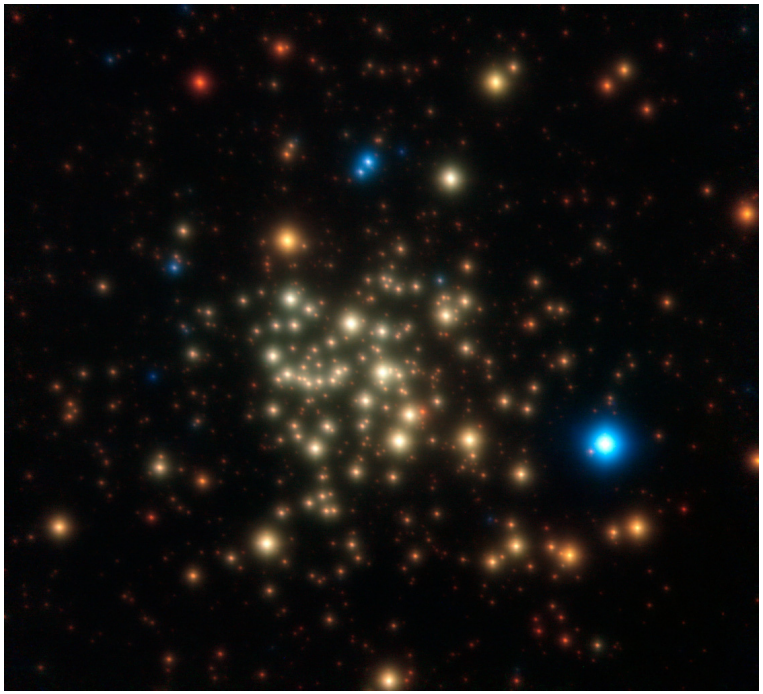
$N = 10k$, $W_0 = 12$, $Q = 0.5$, $b_f = 0.1$, SE, p



The Arches cluster: numerical evolution of dynamical models

The **Arches cluster**: one of the densest and most massive young Galactic star clusters

- age: $t \approx 2.5\,\mathrm{Myr}$
- O-stars: $N \approx 150$ ($m_* > 20\,\mathrm{M}_\odot$)
- mass: $M \gtrsim 10^4\,\mathrm{M}_\odot$ ($\rho \gtrsim 10^5\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$)

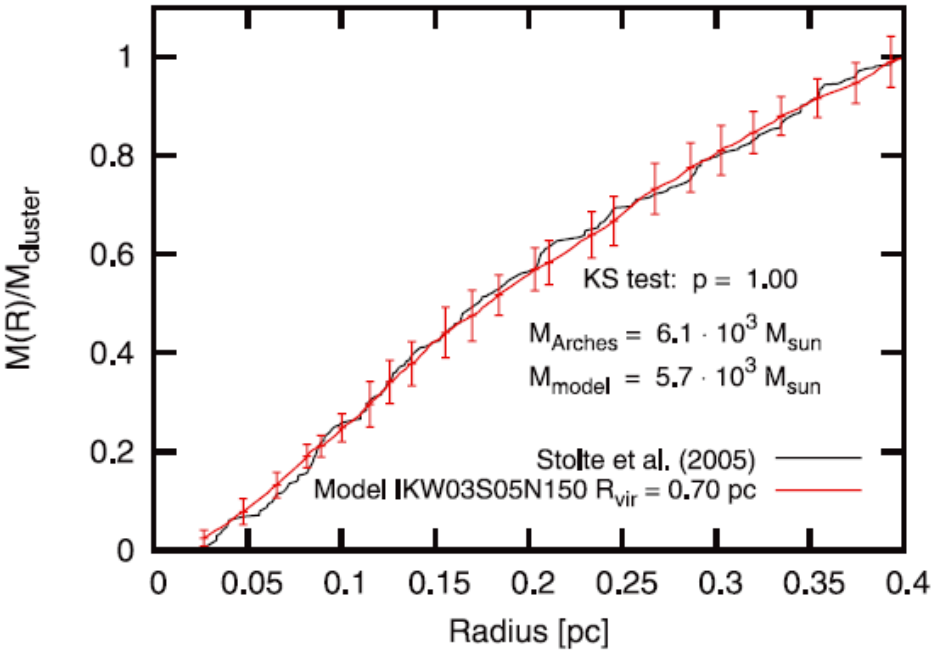


Espinoza et al. (2009)

The Arches cluster: numerical evolution of dynamical models

The **Arches cluster**: one of the densest and most massive young Galactic star clusters

- age: $t \approx 2.5\,\mathrm{Myr}$
- O-stars: $N \approx 150$ ($m_* > 20\,\mathrm{M}_\odot$)
- mass: $M \gtrsim 10^4\,\mathrm{M}_\odot$ ($\rho \gtrsim 10^5\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$)



Models based on parameters estimated by Harfst et al. (2010) and evolved with NBODY6GPU (Aarseth, 2003; Nitadori & Aarseth, 2012).

Two best-fitting (rotating) King models with $W_0 = 3$ and 'standard' IMF from Kroupa (2001):

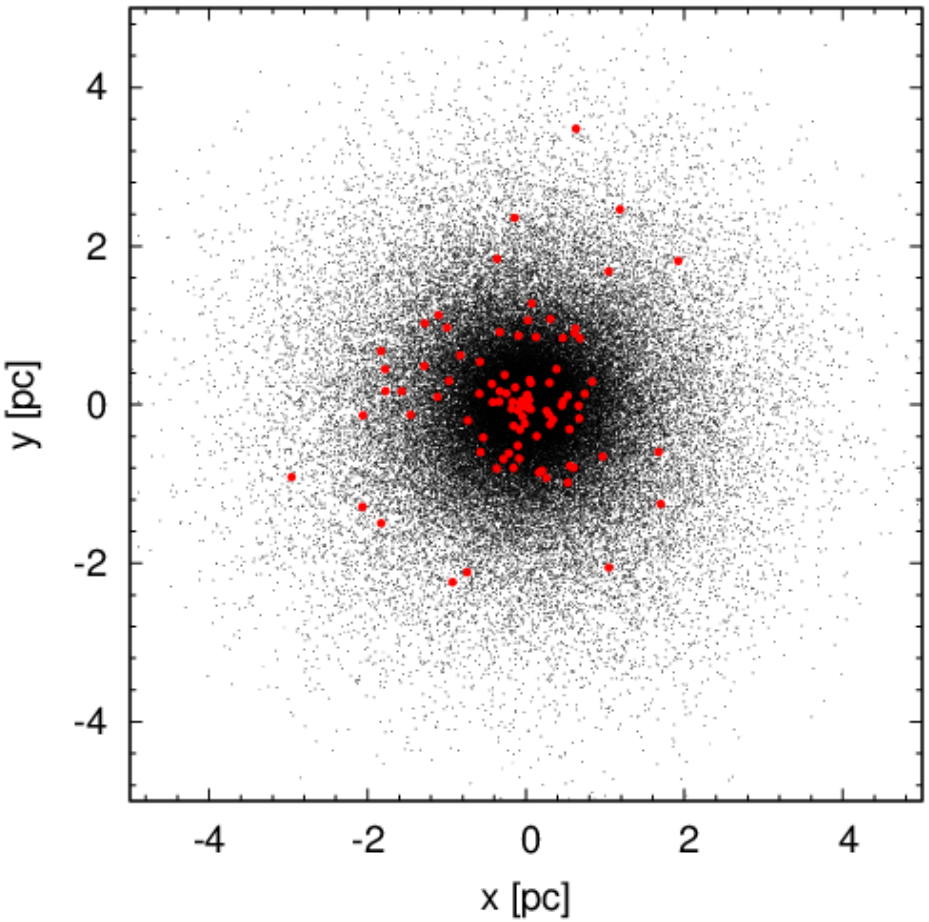
1. **non-rotating** $\mathcal{M}_\mathcal{N}$: $\omega_0 = 0.0$, $N_0 = 66k$ $\quad M_0 = (4.32 \pm 0.06) \cdot 10^4\,\mathrm{M}_\odot$
2. **rotating** $\mathcal{M}_\mathcal{R}$: $\omega_0 = 1.5$, $N_0 = 60k$ $\quad M_0 = (3.87 \pm 0.07) \cdot 10^4\,\mathrm{M}_\odot$

   $\rightarrow$ Note: maximum ω_0 for stable rotation.

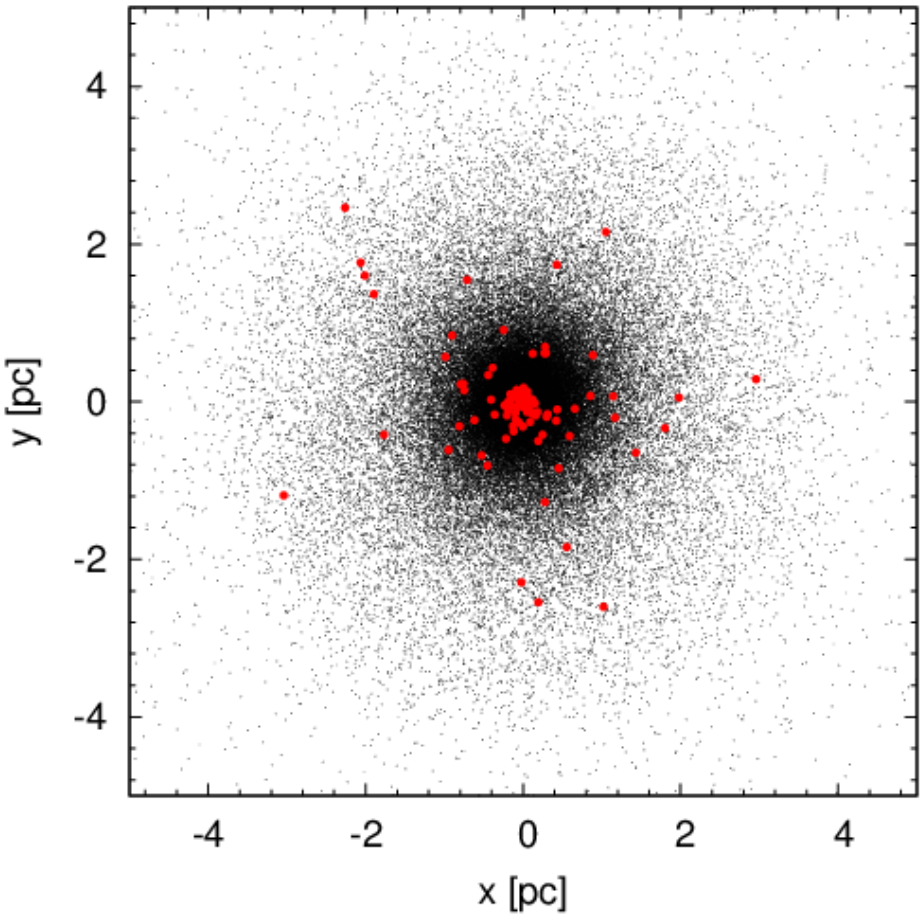
Rapid mass segregation in starburst clusters

Energy equipartition drives strong mass segregation of the 100 most massive stars.

initial

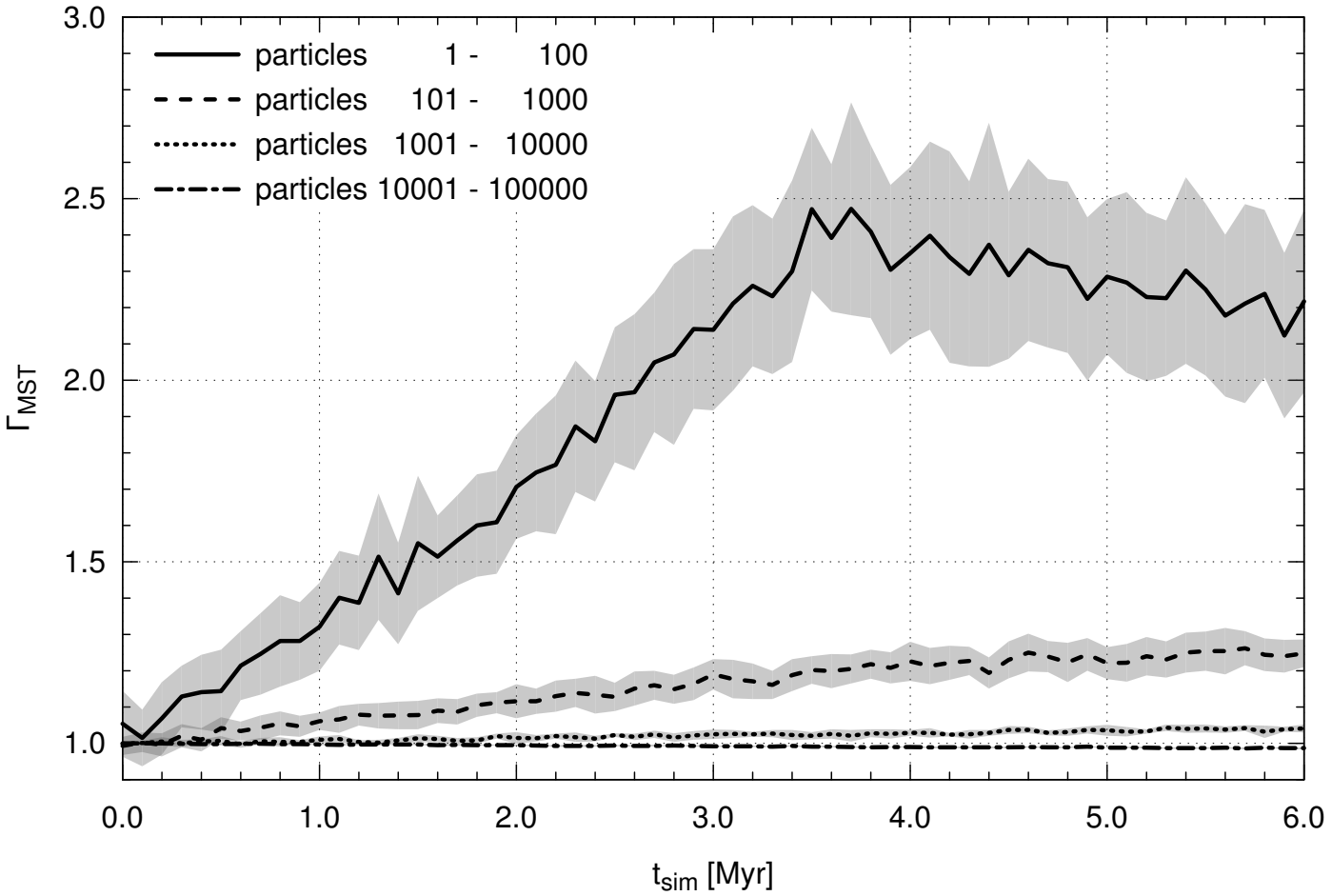


$t = 2\,\mathrm{Myr}$



Rapid mass segregation in starburst clusters

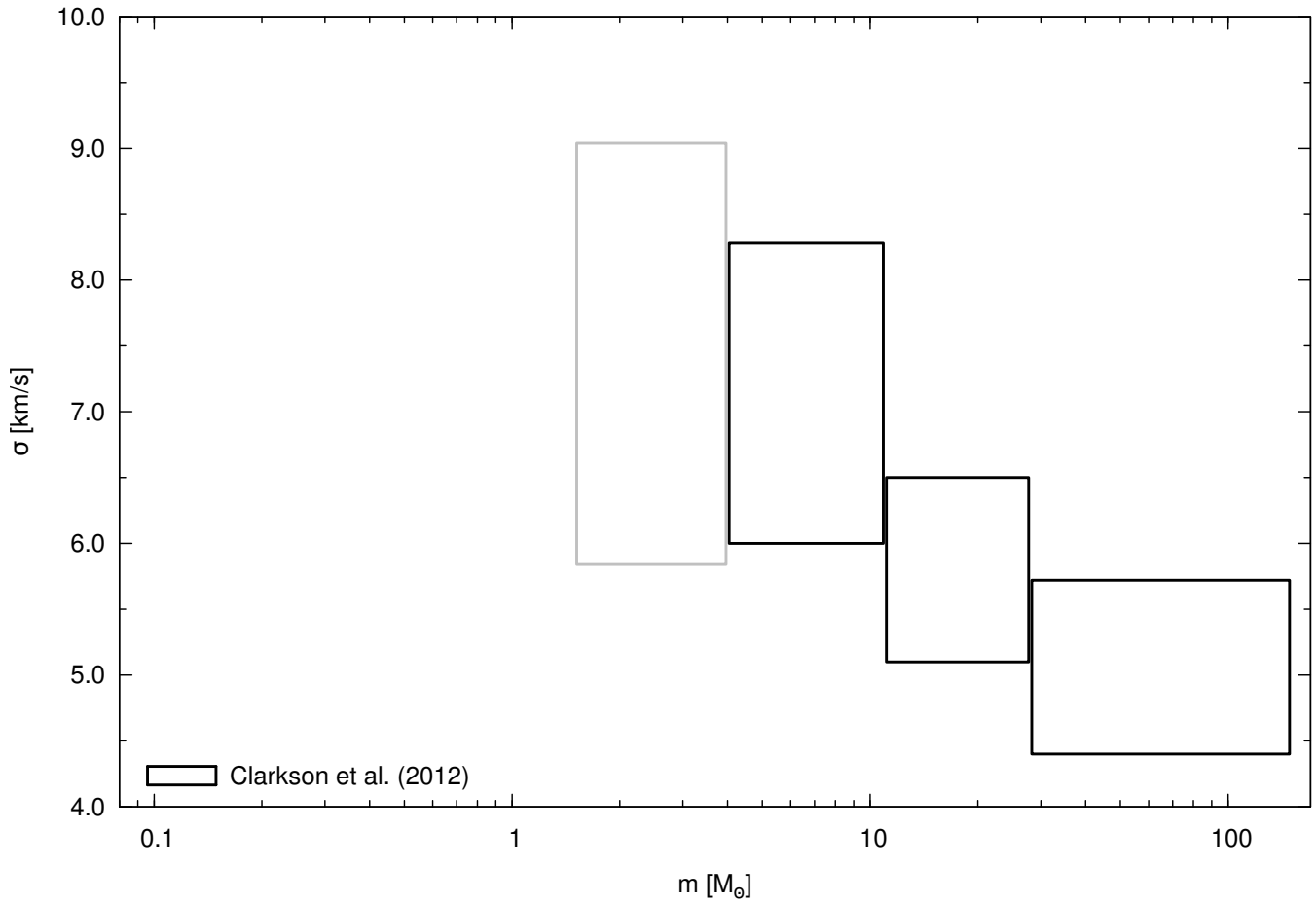
Energy equipartition drives strong mass segregation of the 100 most massive stars.



- Mass segregation measure $\Gamma_{\rm MST}$ (Olczak et al., 2011).

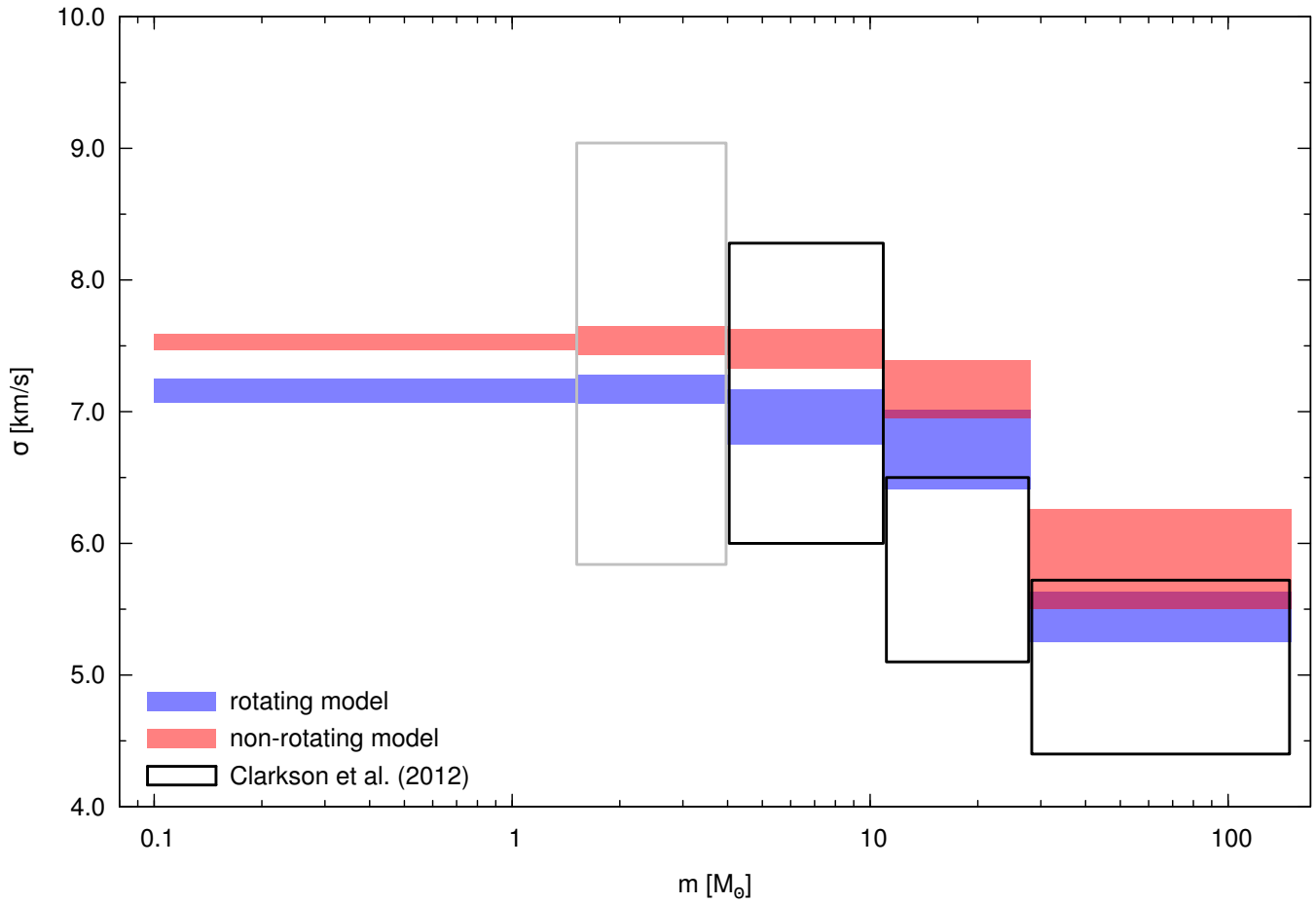
An observational milestone: evidence for mass segregation in velocity space

Mass-dependent velocity dispersion in the cluster core as observed by Clarkson et al. (2012).



An observational milestone: evidence for mass segregation in velocity space

Mass-dependent velocity dispersion in the cluster core as observed by Clarkson et al. (2012).



No evidence for primordial origin of mass segregation.

Summary

Mass segregation is a powerful diagnostic tool

Information about

- star formation process ("primordial mass segregation")
- dynamical evolution

Measuring mass segregation

- Analytic estimates: use $t_{\rm eq}$ rather than $t_{\rm rel}$!
- Efficient numerical measure: $\Gamma_{\rm MST}$ (= Minimum Spanning Tree + geometrical mean)

Mass segregation in (young) star clusters

- Cluster parameters have a strong impact on the evolution of mass segregation:
  $\rightarrow$ speed-up for high concentrations and subvirial initial conditions.
- Stellar evolution strongly affects the signature of mass segregation in young clusters.
- Mass segregation in the Arches cluster in good agreement with dynamical models in **4-dim**.
  $\rightarrow$ No evidence for primordial mass segregation so far.

Aarseth, S. 2003, Gravitational N-body Simulations (Cambridge, Cambridge University Press, 2003, 430 p.)
Allison, R. J., Goodwin, S. P., Parker, R. J., et al. 2009, MNRAS, 395, 1449
Beccari, G., Lützgendorf, N., Olczak, C., et al. 2012, ApJ, 754, 108
Bonnell, I. A. 2000, in ESA Special Publication, Vol. 445, Star Formation from the Small to the Large Scale, ed. F. Favata, A. Kaas, & A. Wilson, 273
Clarkson, W. I., Ghez, A. M., Morris, M. R., et al. 2012, ApJ, 751, 132
Espinoza, P., Selman, F. J., & Melnick, J. 2009, A&A, 501, 563
Gouliermis, D., Keller, S. C., Kontizas, M., Kontizas, E., & Bellas-Velidis, I. 2004, A&A, 416, 137
Harfst, S., Portegies Zwart, S., & Stolte, A. 2010, MNRAS, 409, 628
Hillenbrand, L. A. 1997, AJ, 113, 1733
Hillenbrand, L. A. & Hartmann, L. W. 1998, ApJ, 492, 540
Huff, E. M. & Stahler, S. W. 2006, ApJ, 644, 355
Kroupa, P. 2001, MNRAS, 322, 231
Nitadori, K. & Aarseth, S. J. 2012, MNRAS, 424, 545
Olczak, C., Spurzem, R., & Henning, T. 2011, A&A, 532, A119
Parmentier, G. & Pfalzner, S. 2013, A&A, 549, A132
Raboud, D. & Mermilliod, J.-C. 1998, A&A, 333, 897
Rebull, L. M., Koenig, X. P., Padgett, D. L., et al. 2011, ApJS, 196, 4
Röser, S., Schilbach, E., Piskunov, A. E., Kharchenko, N. V., & Scholz, R.-D. 2011, A&A, 531, A92+
Spitzer, L. J. 1969, ApJ, 158, L139+
Spitzer, L. J. & Hart, M. H. 1971, ApJ, 164, 399
Šubr, L., Kroupa, P., & Baumgardt, H. 2008, MNRAS, 385, 1673