Black Hole Binaries in Nuclear Star Clusters and Gravitational Wave Sources

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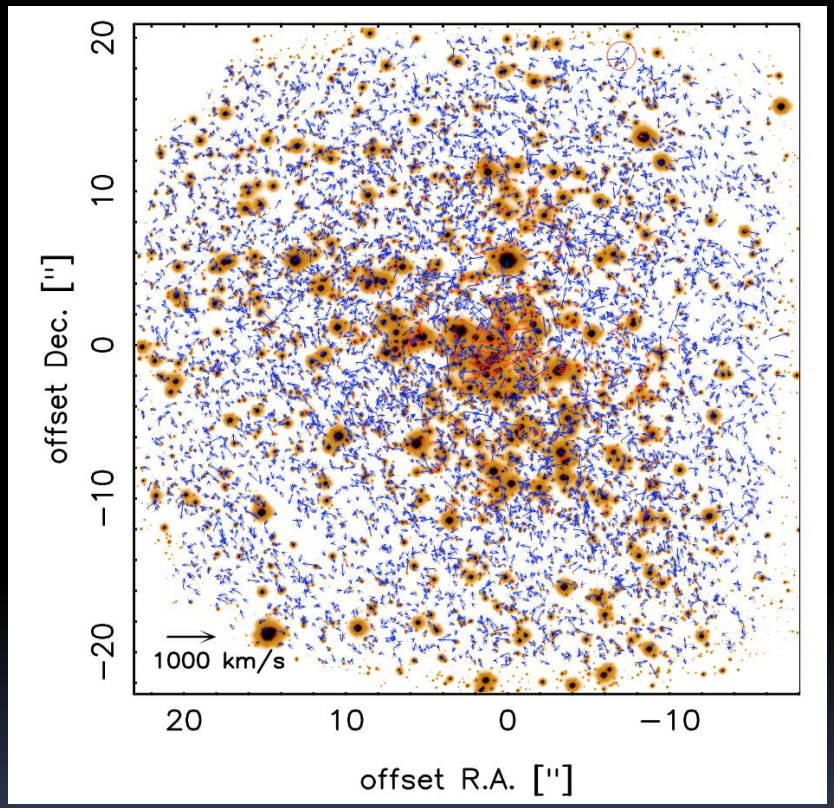
Jongsuk Hong, Hyung Mok Lee
Seoul National University

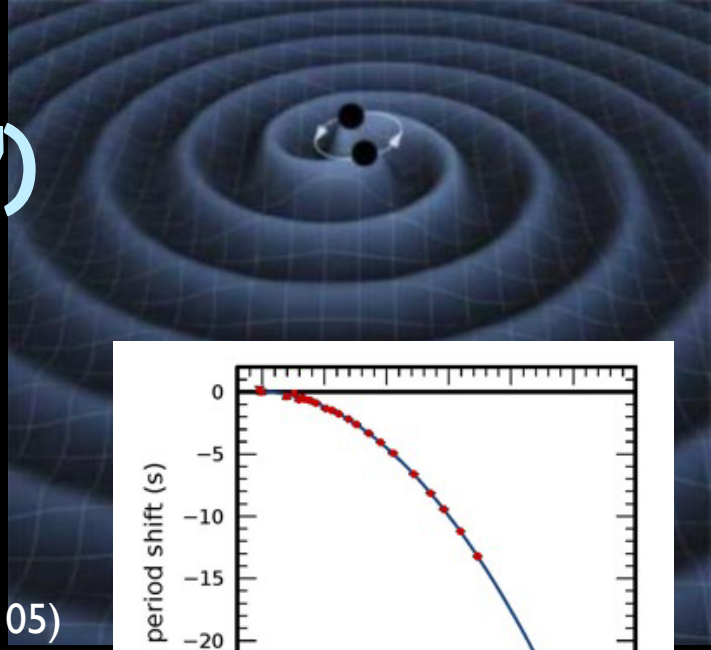
Nuclear Star Clusters (NCs)

- Located at the center of many galaxies
  - Gravitationally bounded in galaxy building blocks (e.g., bulge, disk & halo)
  - 75% in Scd-Sm (Boker et al. 2002), 50% in Sa-Sc (Carollo et al. 1997), 70% in E & S0 (Cote et al. 2006)
- The densest stellar system
  - $M_{NC,MY} \sim 10^5 M_\odot pc^{-3}$ (Schodel et al. 2009)
- Dynamically relaxed
  - Mass segregation; compact stars are centrally-concentrated (Hopman & Alexander 2006; O'Leary et al. 2009)
- Super massive black hole (SMBH) at the center
  - e.g., Sgr A* in the Milky Way
    - $M_{SgrA*} \sim 3\text{-}4 \times 10^6 M_\odot$

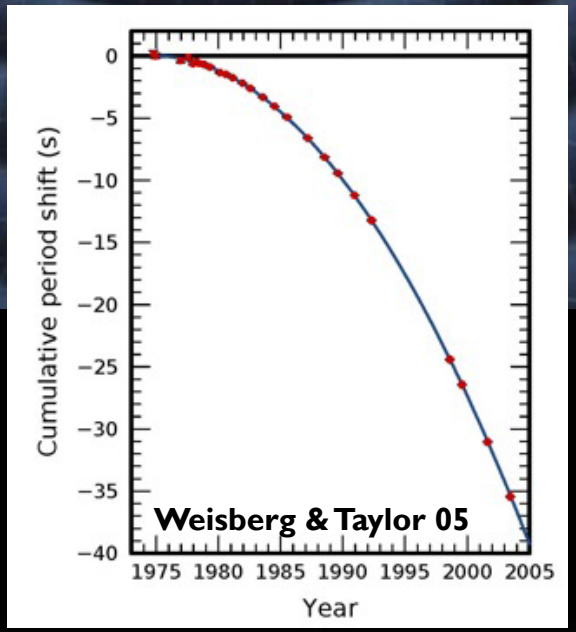


proper motions of stars (Schodel et al. 2009)

Gravitational Wave (GW)



- Propagation of ripples in space-time
  - Predicted by general relativity (Einstein 1916)
- Indirect evidence
  - PSR 1913+16 (Hulse & Taylor 74; Weisberg & Taylor 05)
- Direct detection of GW?
  - Laser interferometers (LIGO, Virgo)



- Astronomical GW sources
  - Core-collapse supernovae (e.g., Mueller & Janka 1997)
  - Spinning neutron stars (e.g., Andersson et al. 2011)
  - Supermassive black holes (e.g., Hopman & Alexander 2005)
  - Compact binary coalescences (e.g., Kalogera et al. 2004)
  - Cosmological gravitational waves (e.g., Ananda et al. 2007)

GW Sources in NCs

- Extreme mass ratio inspirals (EMRI)

  Lightman & Shapiro (1977); Hopman & Alexander (2003, 2005, 2006); Merritt et al. (2011)
  - Compact stars spiraling into the massive black hole
  - low frequency – space band (LISA)

- Compact binary coalescences (CBC)

  Kalogera et al. (2004); Belczynski et al. (2007); O'Leary et al. (2006, 2009); Downing et al. (2011)
  - Gravitational radiation capture (Hansen 1972; Quinlan & Shapiro 1987, 1989)
  - Eccentricity re-distribution by the Kozai effect (Antonini & Perets 2012)
  - Stellar mass black holes binaries (BH-BH, ~10$M_\odot$)
    - One of the main targets for advanced LIGO-Virgo
    - Maximum horizon distance: ~1 Gpc (c.f., ~200 Mpc for NS-NS binary)
    - 1~10% BHs in galactic nuclei (Hopman & Alexander 2006b)

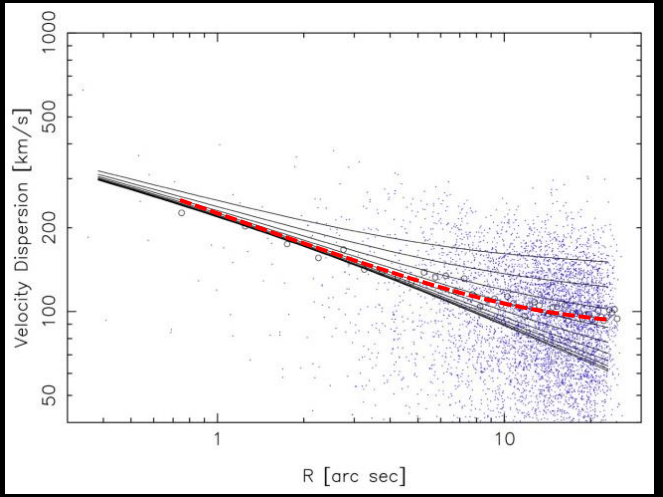
Objective! : estimation of detection rates BH-BH binaries in NCs for Adv. LIGO

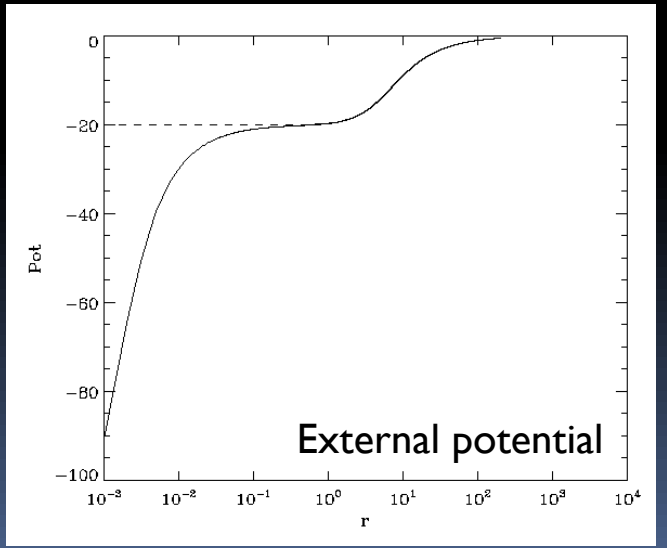
Methods

- NBODY6+GPU (Nitadori & Aarseth 2012)
  - GPU accelerated version of NBODY6
- Initial conditions for NCs
  - Plummer model
  - Equal-mass cases (all 10$M_\odot$ BHs)
- External potential
  - Bulge : Plummer potential
    - Isothermal velocity dispersion (Yoon et al. 2011)
  - SMBH : Point-mass potential
    - Adiabatic growth of MBH (Sigurdsson et al. 1995; Holley-Bockelmann et al. 2002)

$$M(t) = \begin{cases} M_{\rm BH}\left[3\left(\frac{t}{t_{\rm BH}}\right)^2 - 2\left(\frac{t}{t_{\rm BH}}\right)^3\right], & \text{for } t \le t_{\rm BH}\ ; \\ M_{\rm BH}\ , & \text{for } t > t_{\rm BH}\ . \end{cases}$$

Schodel et al. 2009





Simulation Models

Model	$N_{\rm cl}$	$N_{\rm run}$	$M_{\rm MBH}/M_{\rm cl}$	$M_{\rm MBH}/m_*$	$t_{\rm MBH}$	$\varepsilon_{\rm MBH}$	$M_{\rm pl}$	$a_{\rm pl}$
1	10,000	10	0.2	2,000				
2	20,000	5	0.2	4,000				
3	50,000	2	0.2	10,000				
4	100,000	1	0.2	20,000	50	10^{-4}	100	5
5	20,000	5	0.1	2,000				
6	50,000	2	0.1	5,000				
7	100,000	1	0.1	10,000				

MW equivalent, but scale down model

c.f., M_{cl}=1

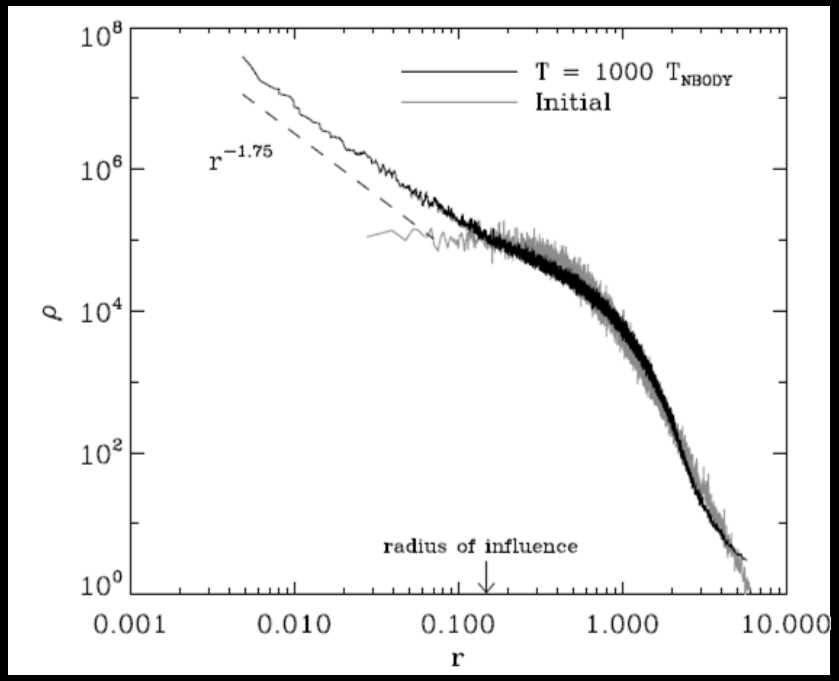
- Different N : scaling to N~$10^{6\text{-}7}$ for NCs
- M_{MBH}/M_{cl} ~ 0.1 for M_{MBH}~$10^6 M_\odot$ (Graham & Spitler 2009)
- t_{MBH} : MBH growth time scale
  - should be larger than dynamical time; $t_{dyn,1/2}$~2.5 (Holley-Bockelmann et al. 2002)
- ε_{MBH} : softening parameter (smooth the singularity of MBH)
  - (c.f., $r_{inf} \equiv M_{MBH}/\sigma^2$~0.1)
- M_{MBH}/M_{pl} = 0.001 / 0.002 (c.f., M_{BH}/M_{bulge} ~ 0.2%; McConnell & Ma 2013)

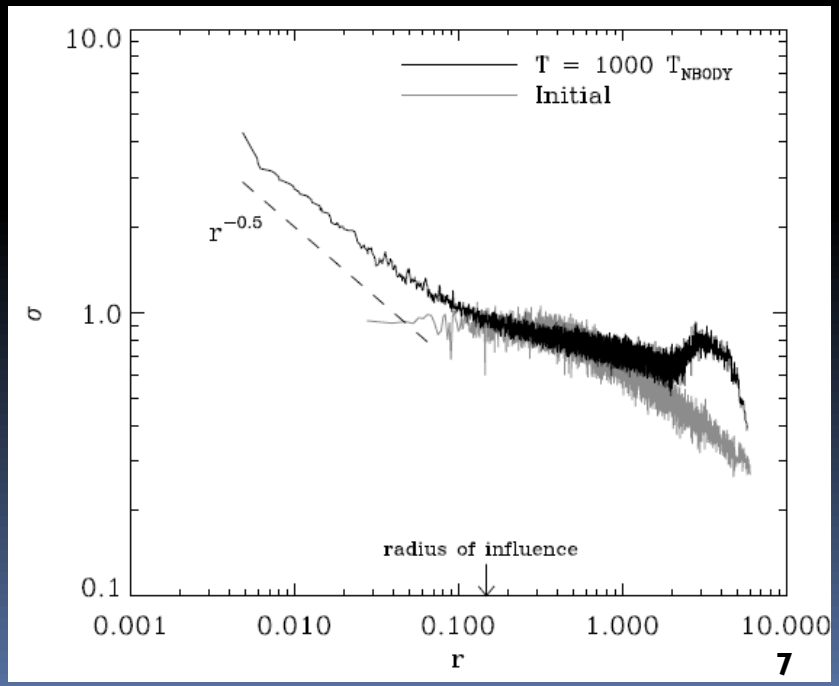
N-body Realizations

- Stellar cusp (Bahcall & Wolf 1976)
  - $\rho(r<r_{inf}) \sim r^{-1.75}$
- Keplerian motion against MBH
  - $\sigma(r<r_{inf}) \sim r^{-0.5}$
- Shallower profiles: MBH wandering (Lin & Tremaine 1980)

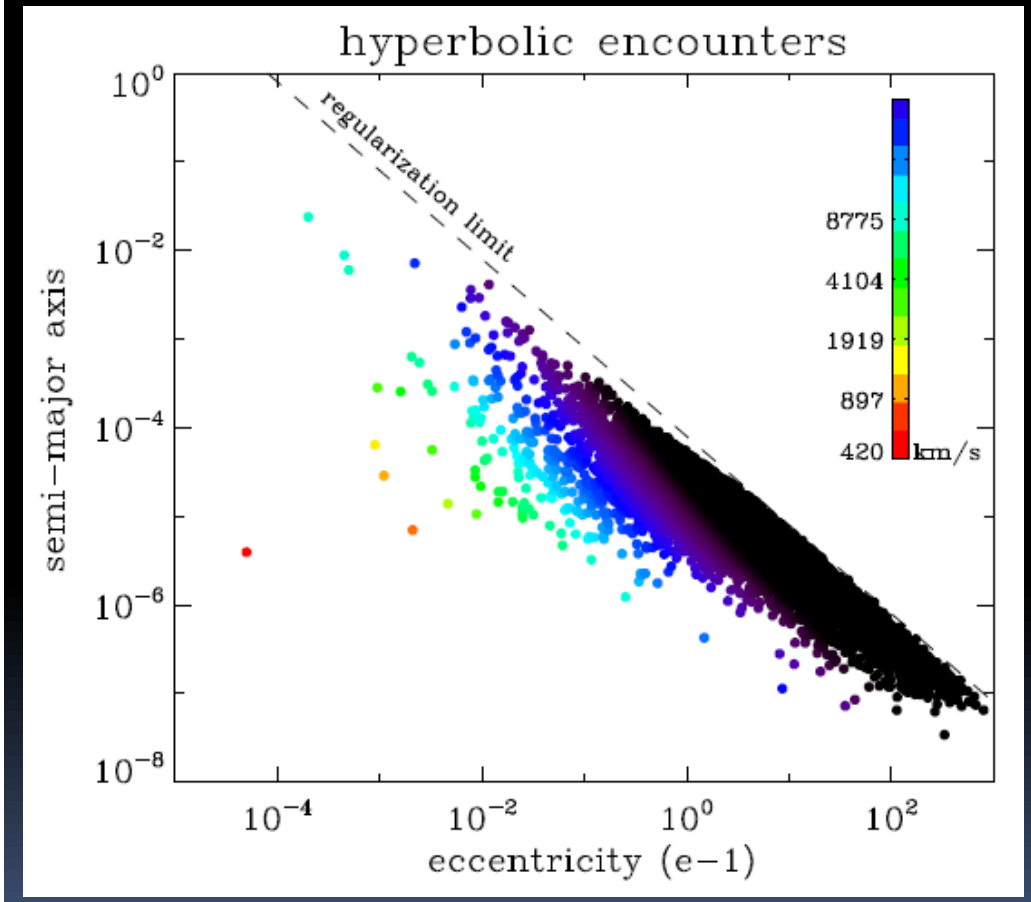
$$r_{\rm wand} \approx 0.92 r_{\rm c} \sqrt{m_*/M_{\rm MBH}},$$

- Flatter velocity dispersion at $r>r_{inf}$: isothermal velocity dispersion





Close Encounters & GR Capture



- At the vicinity of MBH, three-body process is suppressed (Baumgardt et al. 2004a).
- Most of close encounters are hyperbolic encounters.
- When the energy loss by GR is larger than the orbital energy, two BHs become a binary (GR Capture).
- Eccentric / small pericenter

Binary Formation Rates

- GR capture cross section (Quinlan & Shapiro 1987,1989)

$$\Sigma_{\rm cap} = \pi r_{\rm p,max}^2 \left[1 + \frac{2G(m_1+m_2)}{r_{\rm p,max} v_\infty^2}\right] \simeq 17 \frac{G^2 m_1 m_2 \eta^{-5/7}}{c^{10/7} v_\infty^{18/7}}$$

- capture time-scale $(t_{\rm cap} \equiv (\Sigma_{\rm cap} n v)^{-1})$

- GR capture rate in [*r*, *r*+*dr*]
  (O'Leary et al. 2009)

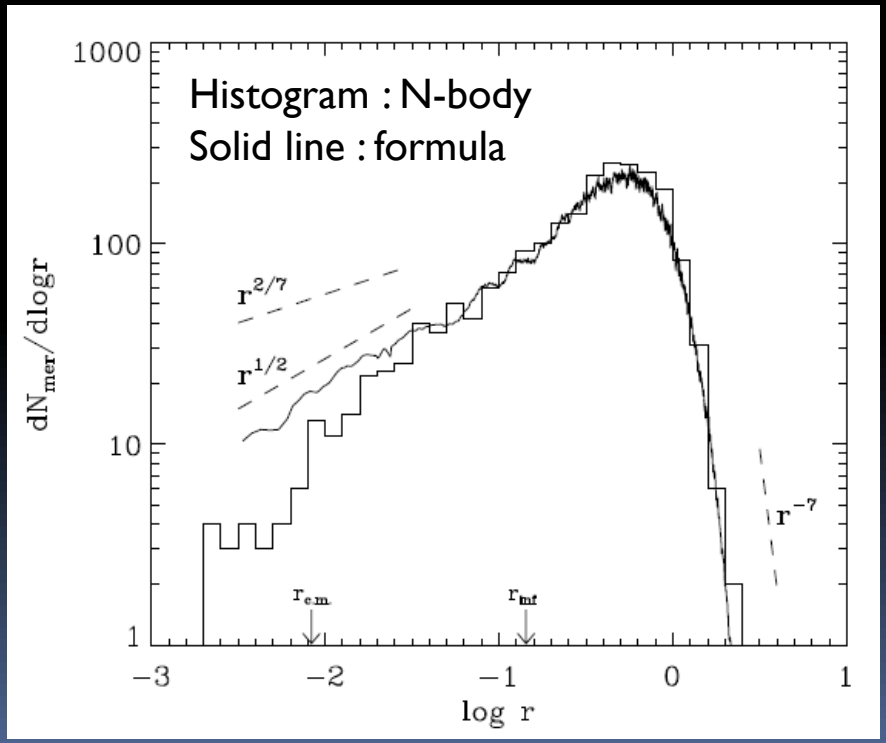
$$\frac{d\Gamma_{\rm cap}}{dr} = 4\pi r^2 n_1(r) n_2(r) \langle \Sigma_{\rm cap} v \rangle,$$

- When m_1=m_2,

$$\frac{d\Gamma_{\rm cap}}{dr} \simeq \frac{1}{2} 87 \frac{G^2 m^2 (1/4)^{-5/7}}{c^{10/7}} r^2 n^2(r) \sigma(r)^{-11/7},$$

- 80% of capture events occur at $r>r_{inf}$.

Radial profile of GR capture rate



Binary Merger Rates

- Formation rates = Merger rates

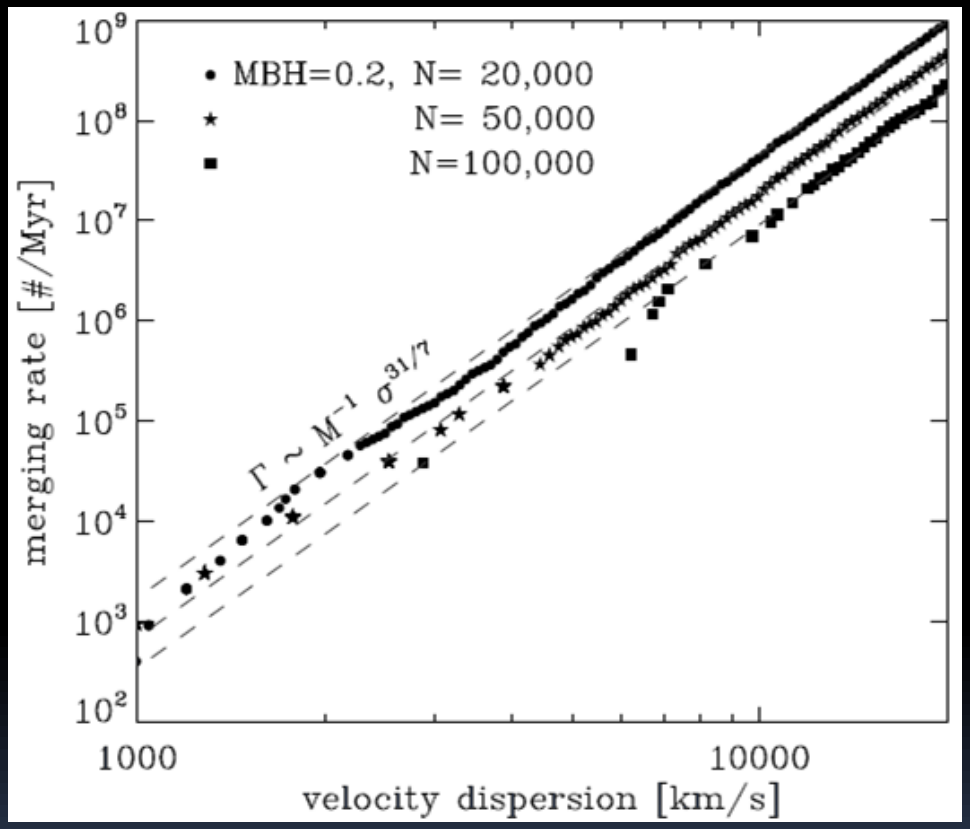
  T_{mer} « Myr (Eccentric, small pericenter)

- Overall merger rates

$$\begin{aligned}\Gamma_{mer} &\approx m^2 \cdot N \cdot \tilde{n} \cdot \sigma_*^{-11/7} \\ &\approx M^2 \cdot V^{-1} \cdot \sigma_*^{-11/7} \\ &\approx M^2 \cdot (M^{-1}\sigma_*^2)^3 \cdot \sigma_*^{-11/7} \\ &\approx M^{-1} \cdot \sigma_*^{31/7},\end{aligned}$$

- Systemic velocity dispersion (McConnell & Ma 2013)

$$\sigma_*^2 \equiv \frac{\int_{r_{\rm inf}}^{r_{\rm half}} 4\pi\sigma^2(r)\rho(r)r^2dr}{\int_{r_{\rm inf}}^{r_{\rm half}} 4\pi\rho(r)r^2dr},$$



- Merger rates per NC

$$\Gamma_{\rm mer} \approx 2.06\times10^{-4}{\rm Myr}^{-1}\left(\frac{M_{\rm MBH}}{3.5\times10^6 M_\odot}\right)^{-1}\left(\frac{\sigma_*}{75{\rm km/s}}\right)^{31/7}, \text{ for } M_{\rm MBH}=0.2M_{\rm tot},$$

Merger Rates to Detection Rates

- Merger rates as a function of M_{MBH} from M_{MBH}-σ relation (Tremaine et al. 2002)

$$M_{\rm MBH} \approx 1.3 \times 10^8 M_\odot (\sigma_*/200{\rm kms}^{-1})^4 \quad \Rightarrow \quad \Gamma \sim M_{\rm MBH}^{3/28}$$

- Merger rate density from MBH density function (Aller & Richstone 2002)

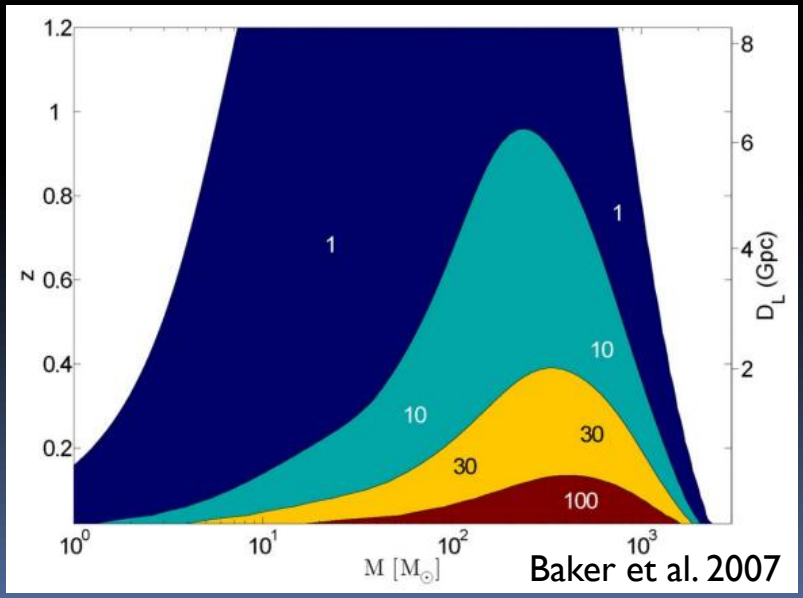
$$\frac{dn_{\rm MBH}}{dM_{\rm MBH}} = c_\bullet \left(\frac{M_{\rm MBH}}{M_\bullet}\right)^{-\alpha} e^{-M_{\rm MBH}/M_\bullet} \quad \Rightarrow \quad \mathcal{R}_{\rm mer} = \int_{M_{\rm l}}^{M_{\rm u}} \Gamma_{\rm mer}(M_{\rm MBH}) \frac{dn_{\rm MBH}}{dM_{\rm MBH}} dM_{\rm MBH}$$

$$(c_\bullet, M_\bullet, \alpha) = (3.2 \times 10^{-11} M_\odot^{-1} {\rm Mpc}^{-3}, 1.3 \times 10^8 M_\odot, 1.25)$$

- Detection rate from volume integration
  (O'Leary et al. 2006; Belczynski et al.2007; Downing et al.2011)

$$\mathcal{R}_{\rm det} = \mathcal{R}_{\rm mer} \int \frac{4\pi r(z)^2}{1+z} \frac{dr}{dz} dz,$$

  - $(1+z)^{-1}$: cosmological time dilation
  - up to the maximum horizon distance D_h
    (Baker et al. 2007; Reisswig et al. 2009; Abadie et al. 2010)



Baker et al. 2007

Detection Rates for Adv. LIGO

Models	$M_{\rm MBH}/M_{\rm cl}$	$\Gamma_{\rm mer,MW}$ ($\rm Myr^{-1}$)	$\mathcal{M}^a_{\rm BH}$	$D_{\rm h}$ (Mpc)	$\mathcal{R}^b_{\rm det,l}$ ($\rm yr^{-1}$)	$\mathcal{R}^c_{\rm det,re}$ ($\rm yr^{-1}$)	$\mathcal{R}^d_{\rm det,h}$ ($\rm yr^{-1}$)
1-4	0.2	3.33×10^{-4}	0.44	986[e]	0.06	0.44	2.99
				~1100[f]	0.09	0.64	4.32
				~1900[g]	0.27	2.00	13.5
5-7	0.1	1.39×10^{-4}	0.44	986[e]	0.02	0.19	1.25
				~1100[f]	0.04	0.27	1.80
				~1900[g]	0.11	0.83	5.62

[e] Abadie et al. 2010
[f] Baker et al. 2007
[g] Reisswig et al. 2009

- Uncertainties (~100):
  - Dynamical evolution of clusters: factor of 2~3
  - Variance of the number of stars (O'Leary et al. 2009): factor of ~10
  - Mass range of MBH: factor of 2~3
- BH-BH Detection rates: 0.02-20 $\rm yr^{-1}$
  - 1-1000 $\rm yr^{-1}$ (Fokker-Planck simulations for GNi; O'Leary et al. 2009)
  - c.f., 1-2 $\rm yr^{-1}$ (field population synthesis; Belczynski et al. 2007), 15-40 $\rm yr^{-1}$ (dynamical formation and hardening in GCs; Monte-Carlo simulation, Downing et al. 2011; N-body simulation, Banerjee et al. 2010, Bae et al. in prep.)

Summary

- We have generated N-body realizations for nuclear star clusters by using
  - external potential of bulge
  - adiabatically growing point-mass potential of MBH,

  and our realizations are equivalent to the theories and observations.

- The expected GW detection rate of BH-BH binaries in NCs for Adv. LIGO is 0.02-20 yr^{-1} with uncertainties:
  - dynamical evolution of clusters
  - variance of the number density
  - mass range of MBH