Post-Newtonian N-Body Simulations

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Astrophysical Scenarios

Self-Consistent Modelling

Recent Improvements

Some Results

Discussion

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PN Scenarios

Unperturbed binaries	BH or NS
Globular clusters	BH or NS + N^*
Galactic centres	IMBH + BH + N^*
Supermassive systems	SMBH + IMBH + N^*

Energy considerations $\quad \dfrac{m_1 m_2}{2a^*} = \kappa |E_{\rm tot}|, \quad E_{\rm tot} = -0.25$

Super-hard binary $\quad m_1 = 10\bar{m}, \quad \kappa = 0.1, \quad a^* = \dfrac{2000}{N^2}$

Schwarzschild radius $\quad R_{\rm Sch} = \dfrac{2M}{c^2} = \dfrac{4}{9} \times 10^{-10} \kappa V^{*2}$

GR radiation time-scale $\quad t_{\rm GR} \propto \dfrac{c^5 a^5}{m_1^2}(1-e^2)^{7/2}, \quad c = \dfrac{3 \times 10^5}{V^*}$

Full Simulations

Initial conditions $N = 1\times10^5$, Kroupa IMF in $0.1 - 50 M_\odot$

Plummer & tides $R_h = 1.0$ pc, $V^* = 16$ km/s

Stellar evolution BH/NS formation above $20/8\, M_\odot$, # 150/680

Velocity kicks Maxwellian $\sigma = 2V^*$, $v_\infty \simeq (1-2)V^*$

N-body units $G = 1,\ \bar{m} = 1/N,\ E = -\frac{1}{4},\ \bar{v}^2 = \frac{1}{2},\ R_\mathrm{h} \simeq 1$

Hard binary $\frac{m^2}{2a} \simeq \frac{1}{2}\bar{m}\bar{v}^2, \quad \Rightarrow a_\mathrm{hard} \simeq 2/N$

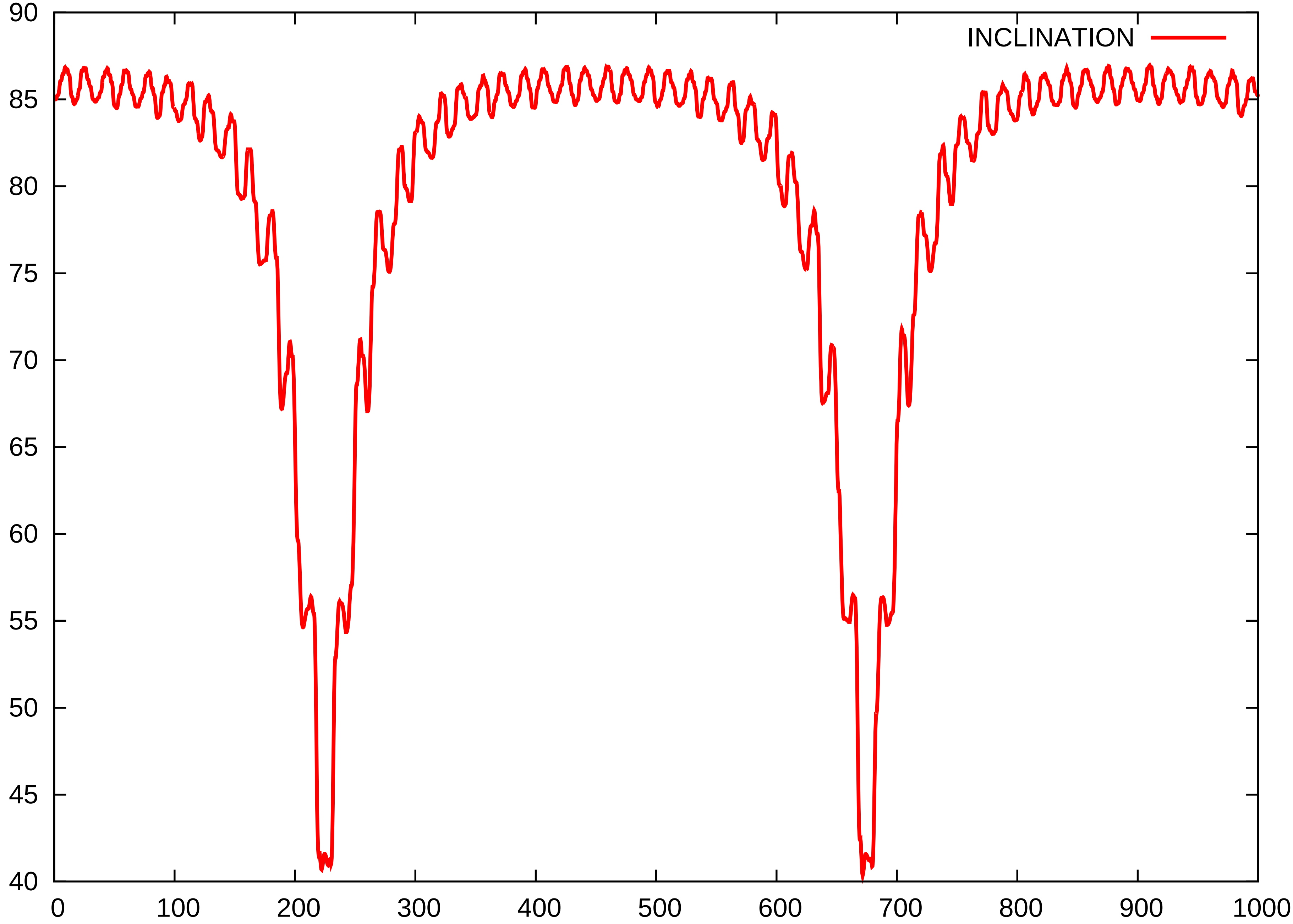
Hard limit $-\frac{m^2}{2a} = E, \quad \Rightarrow a_0 = 2/N^2, \quad \text{or } 2\times10^{-10}$

Dynamics $m_\mathrm{bh} \simeq 20\ M_\odot, \quad E_\mathrm{b} = 0.01E, \quad a \simeq 2\times10^{-5}$

Relativity $R_\mathrm{co} = \frac{8(m_1+m_2)}{c^2}, \quad c = \frac{3\times10^5}{V^*}, \quad R_\mathrm{co} \simeq 2\times10^{-11}$

PN condition $a(1-e) \simeq 1\times10^3 R_\mathrm{co}, \quad \Rightarrow e > 0.999$

Time-scale $a = 2\times10^{-5},\ e = 0.999,\ m = 20\,\mathrm{M}_\odot,\ \tau_\mathrm{GR} \simeq 2\,\mathrm{Myr}$



INCLINATION
90
85
80
75
70
65
60
55
50
45
40
0
100
200
300
400
500
600
700
800
900
1000

PN Decision-Making

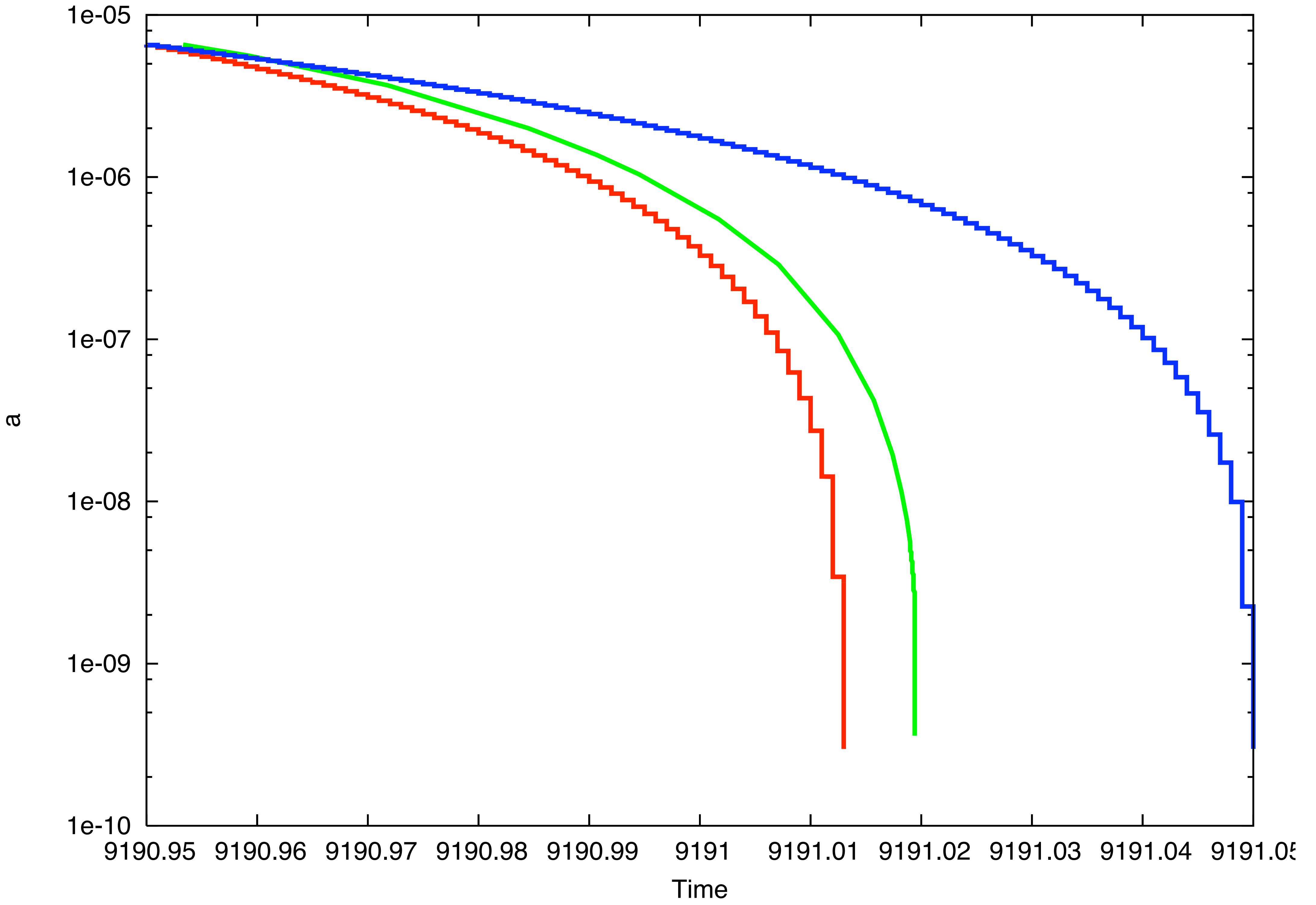
Equation of motion	$\frac{d^2\mathbf{r}}{dt^2} = \frac{M}{r^2}\left[(-1+A)\frac{\mathbf{r}}{r} + B\mathbf{v}\right]$
GR radiation time-scale	$t_{\rm GR} = \frac{5}{64}\frac{c^5\, g(e)\, a^4}{X(1+X)\, m_1^3}, \quad c = \frac{3\times 10^5}{V^*}$
	$g(e) \simeq \frac{(1-e^2)^{7/2}}{4.35}, \quad X = \frac{m_2}{m_1}$
Graduated GR effect	Blanchet & Iyer 2003
	$t_{\rm GR} < 500, \ 50, \ 1$
	$\mathrm{IPN} = 1, \ 2, \ 3$
Coalescence	$R < 4R_{\rm Sch} = \frac{8\,M}{c^2}$
Alternative merging	$\mathrm{IPN} = 3, \quad N = 2, \quad N_{\rm p} = 0$
	$\mathrm{IPN} \geq 2, \quad a\,(1-e) < R_{\rm Sch}$
	$\mathrm{IPN} = 3, \quad N = 3, \quad a_1\,(1-e_1) > 100\,a$
Energy check	$E_{\rm tot} - \int \mathbf{P}_{\rm GR}\cdot\mathbf{v}\,dt = \mathrm{const}$

Unperturbed GR Orbit

Compact objects	$\max\,(K_1^*, K_2^*) \;>\; 12$
Derivatives	$\dot{a} = -\dfrac{64 M_1 M_2 (M_1 + M_2)}{5 c^5 a^3 (1 - e^2)^{7/2}}$
	$\dot{e} = -\dfrac{304 e M_1 M_2 (M_1 + M_2)}{15 c^5 a^4 (1 - e^2)^{5/2}} g(e)$
Einstein shift	$\Delta\omega = \dfrac{6\pi (M_1 + M_2)}{a c^2 (1 - e^2)}$
Safety condition	$\Delta t = \min\,\left(\dfrac{0.01 a}{\dot{a}}, \Delta t_0\right)$
KS rotation	$\theta = \dfrac{\Delta\omega \Delta t}{T_{\rm K}}$, rotate by $\theta/2$
New elements	$a_1 = a + \dot{a}\Delta t, \quad e_1 = \max\,(e + \dot{e}\Delta t, 0)$
Energy updating	$\Delta E = \mu (H_{\rm old} - H_{\rm new})$
Modify KS orbit	Shrink, $e = $ const; decrease, $H = $ const
Coalescence check	$a_1 < \dfrac{8 (M_1 + M_2)}{c^2}$

Unperturbed PN

PN condition	max $(k^*) \geq 13$
Derivatives	$\dot{a},\ \dot{e},$ Peters 1964
Time-scale check	$t_{\rm GR} < 500$
Transformation	$\mathbf{U},\ \mathbf{U}' \Rightarrow$ peri
Unperturbed apo	$\mathbf{U}_{\rm apo} = \mathbf{U}'_{\rm peri}/\omega, \quad \mathbf{U}'_{\rm apo} = -\omega \mathbf{U}_{\rm peri},$ $\omega = (-h/2)^{1/2}$
Rotation of orbit	$\theta = \Delta\omega \Delta t / T_{\rm K}$
New elements	$a_1 = a + \dot{a}\Delta t, \quad e_1 = e + \dot{e}\Delta t$
KS corrections	$(i)\ \ h = \text{const}, \quad (ii)\ \ e = \text{const}$
Perturbed case	Initialize KS polynomials & time-step
Unperturbed case	Continue with current T_0 & Δt


1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
a
9190.95
9190.96
9190.97
9190.98
9190.99
9191
9191.01
9191.02
9191.03
9191.04
9191.05
Time

PN Energy Loss

Equation of motion

$$\mathbf{a} = \mathbf{a}_N + \mathbf{a}_{\rm pn} = -\frac{m}{r^3}\mathbf{r} + \frac{m}{r^2 c^5}\left(A\frac{\mathbf{r}}{r} + B\mathbf{v}\right)$$

Energy change

$$\Delta E_{\rm GR} = \mu\left(\mathbf{a}_{\rm pn}\cdot\mathbf{v}\,\Delta t + \frac{1}{2}(\mathbf{a}_{\mathbf{pn}}\cdot\mathbf{a} + \dot{\mathbf{a}}_{\rm pn}\cdot\mathbf{v})\Delta t^2\right)$$

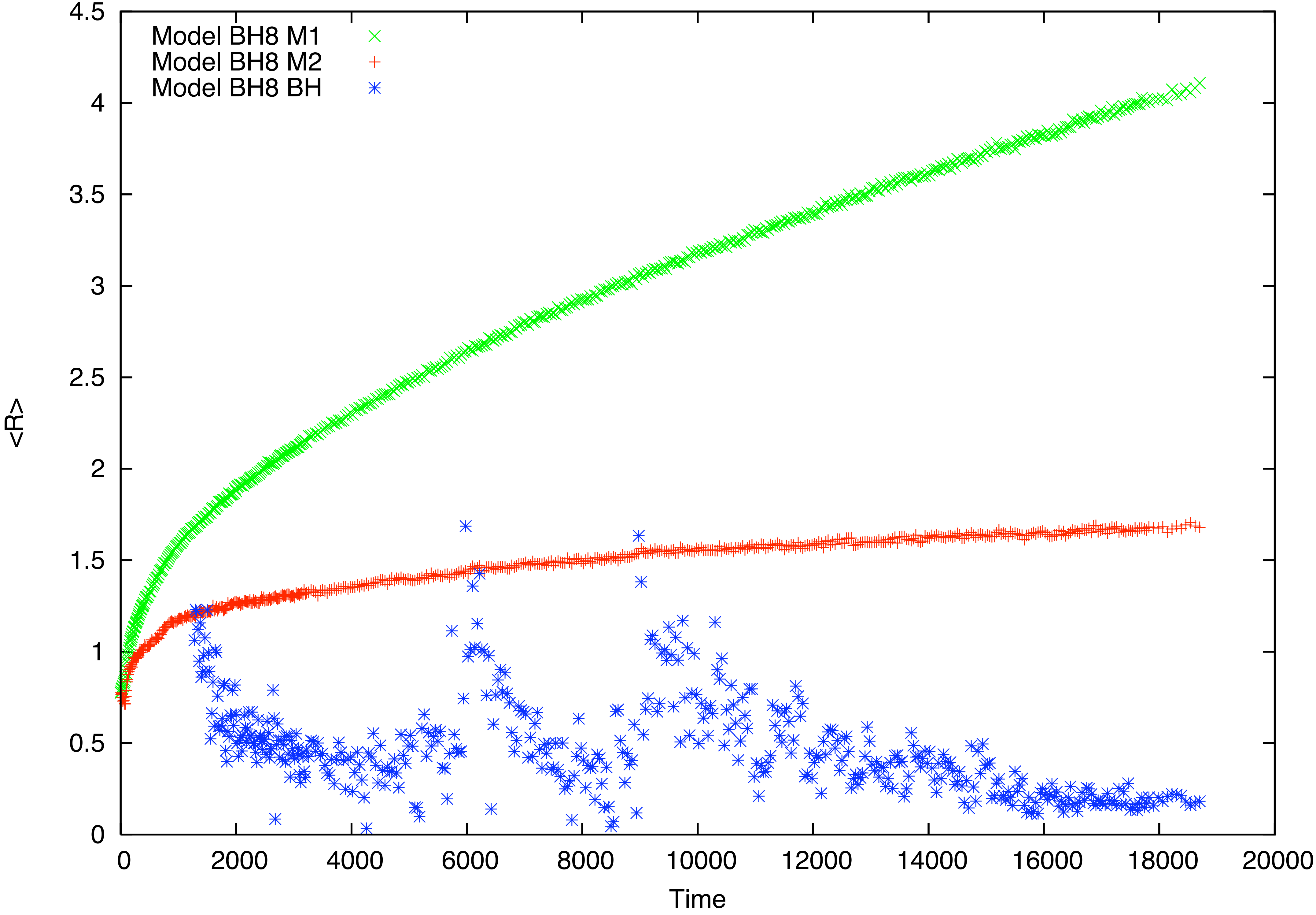
Derivatives $\dot{A}$, $\dot{B}$, save $\dot{\epsilon}_0$, $\ddot{\epsilon}_0$

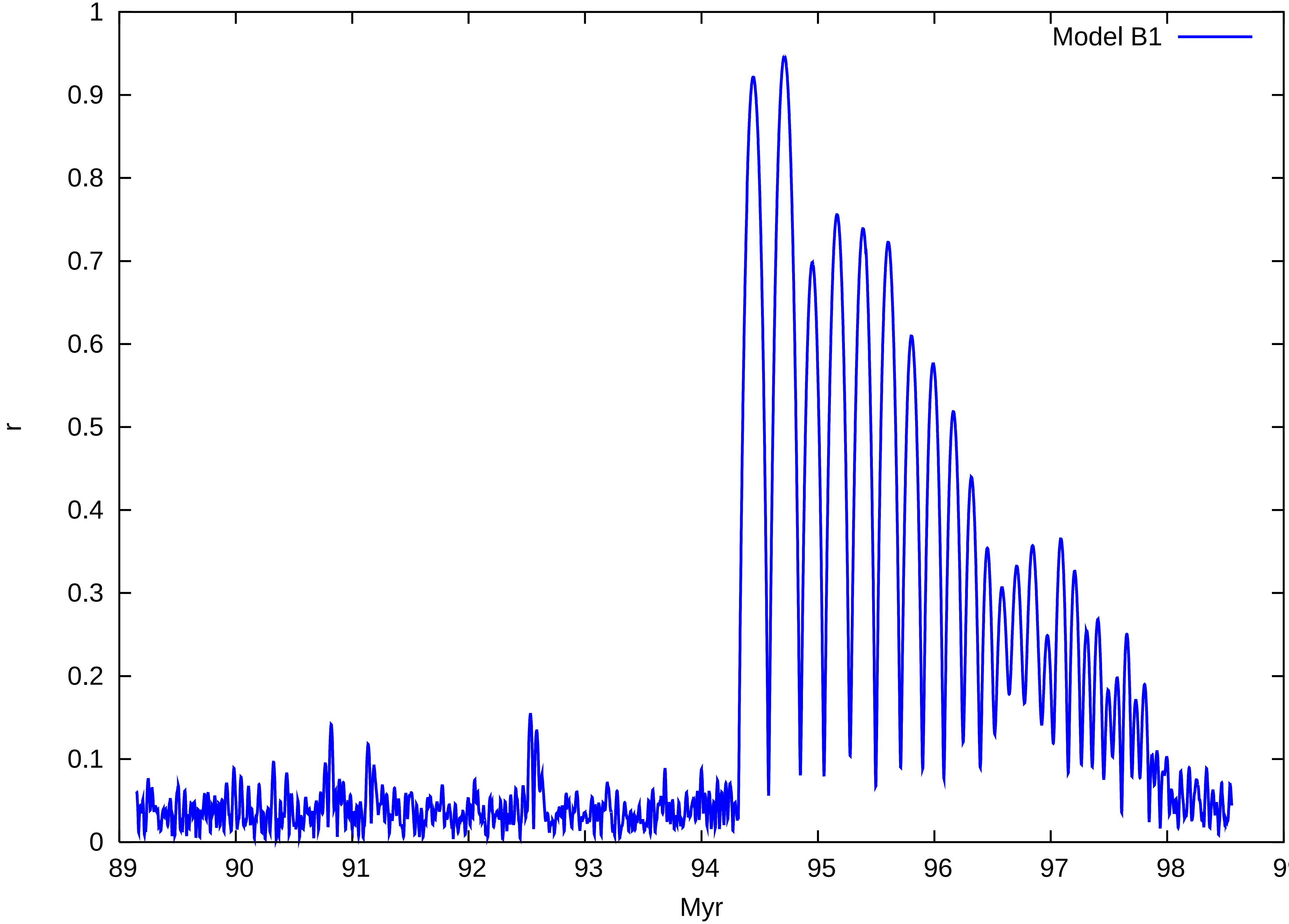
Hermite solution Second term as $\ddot{\epsilon}$

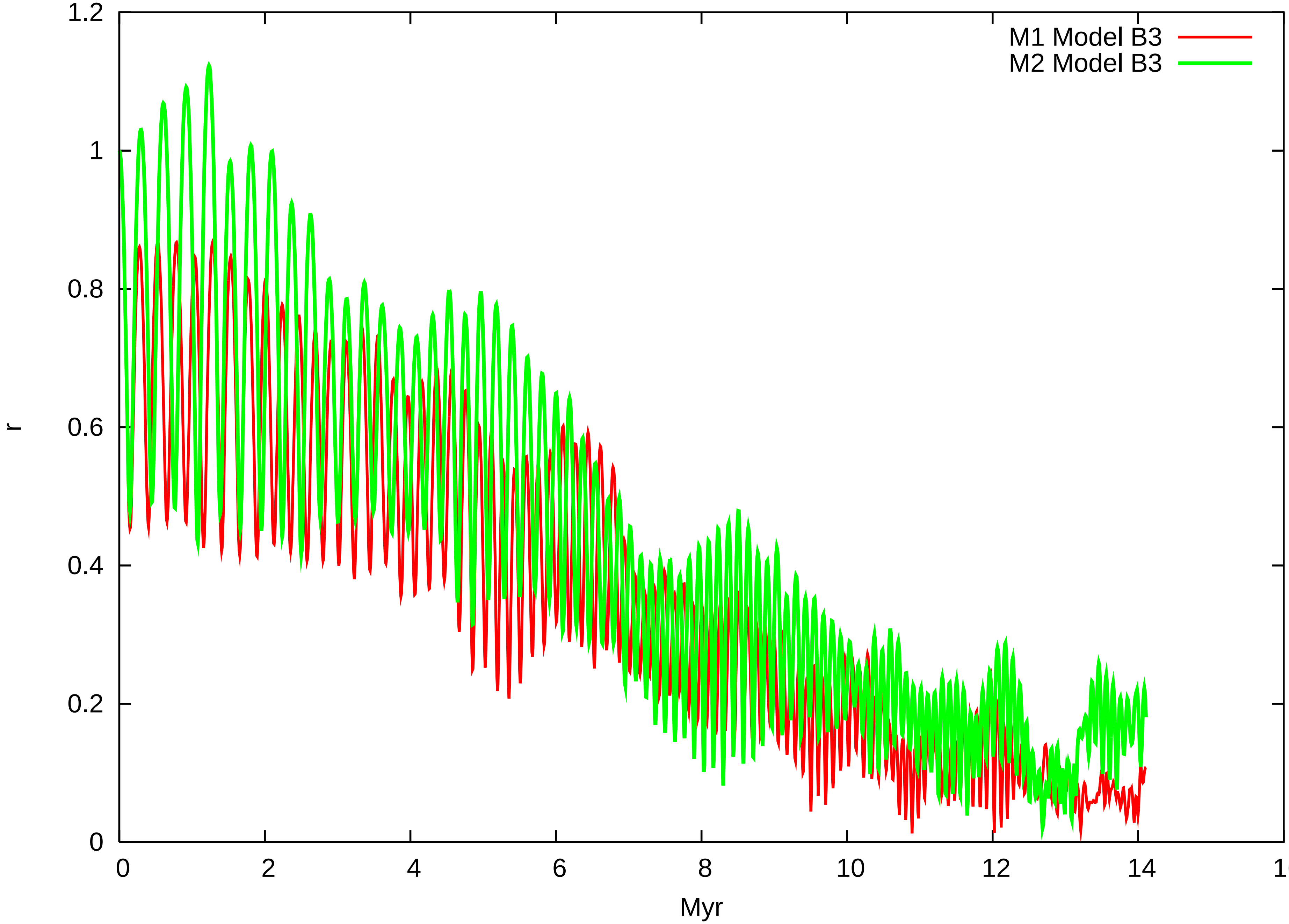
Fourth-order solution

$$\Delta\epsilon = \frac{1}{2}(\dot{\epsilon}_0 + \dot{\epsilon})\,\Delta t + \frac{1}{12}(\ddot{\epsilon}_0 - \ddot{\epsilon})\,\Delta t^2$$

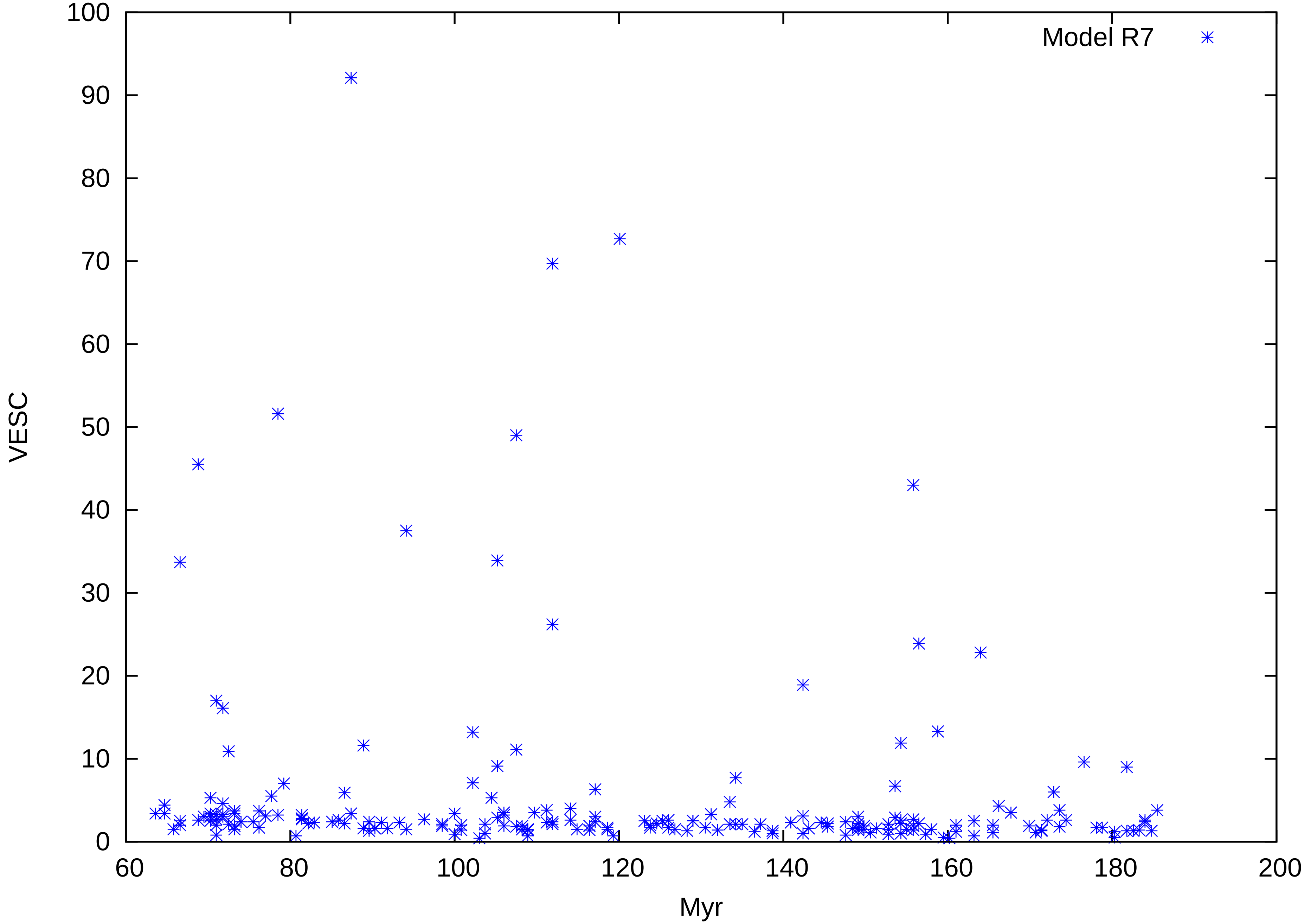
Comparison Agreement with PN 2.5 of Peters 1964


Model BH8 M1
Model BH8 M2
Model BH8 BH
<R>
Time
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000

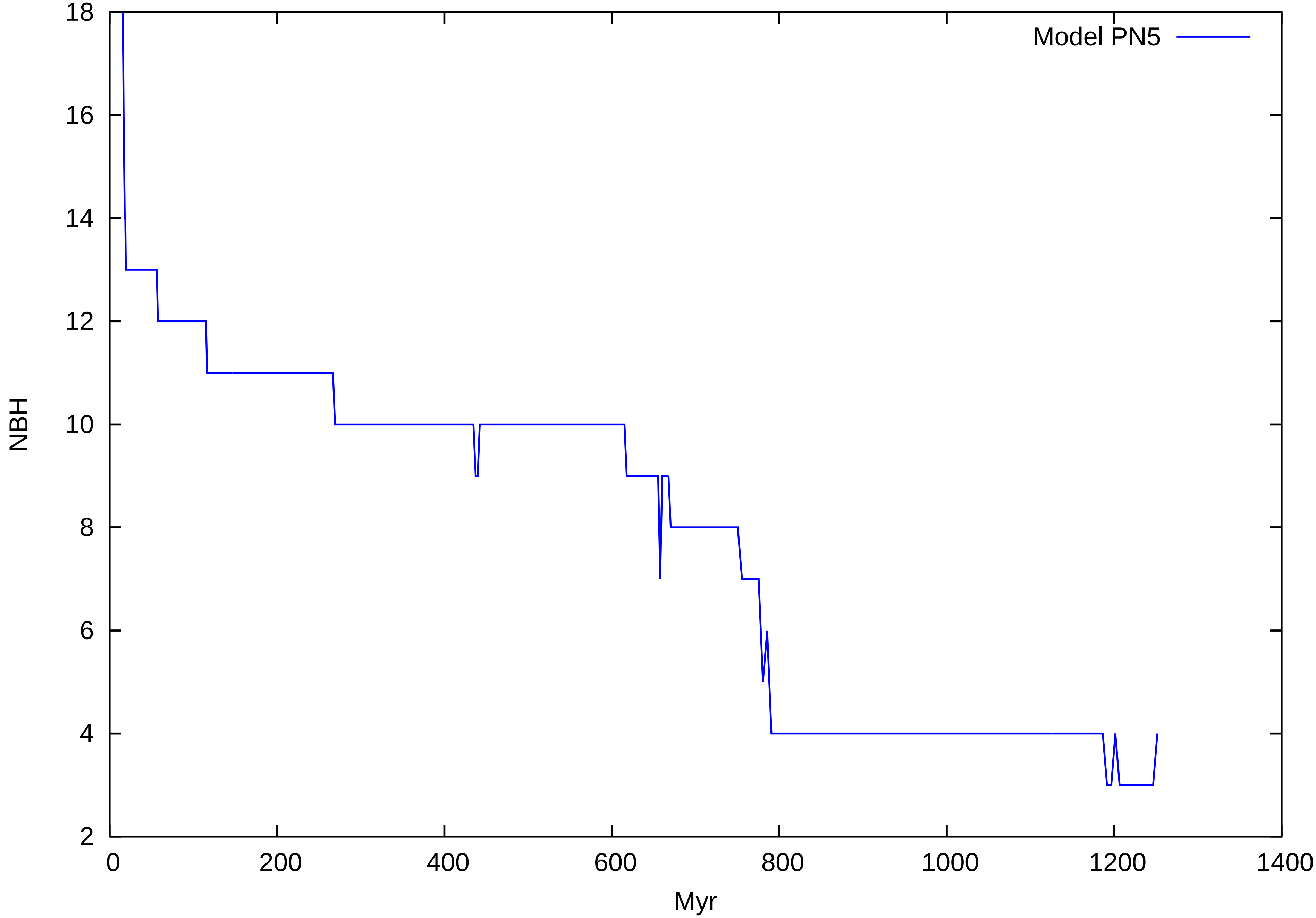

Model B1
r
Myr
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
89
90
91
92
93
94
95
96
97
98
99



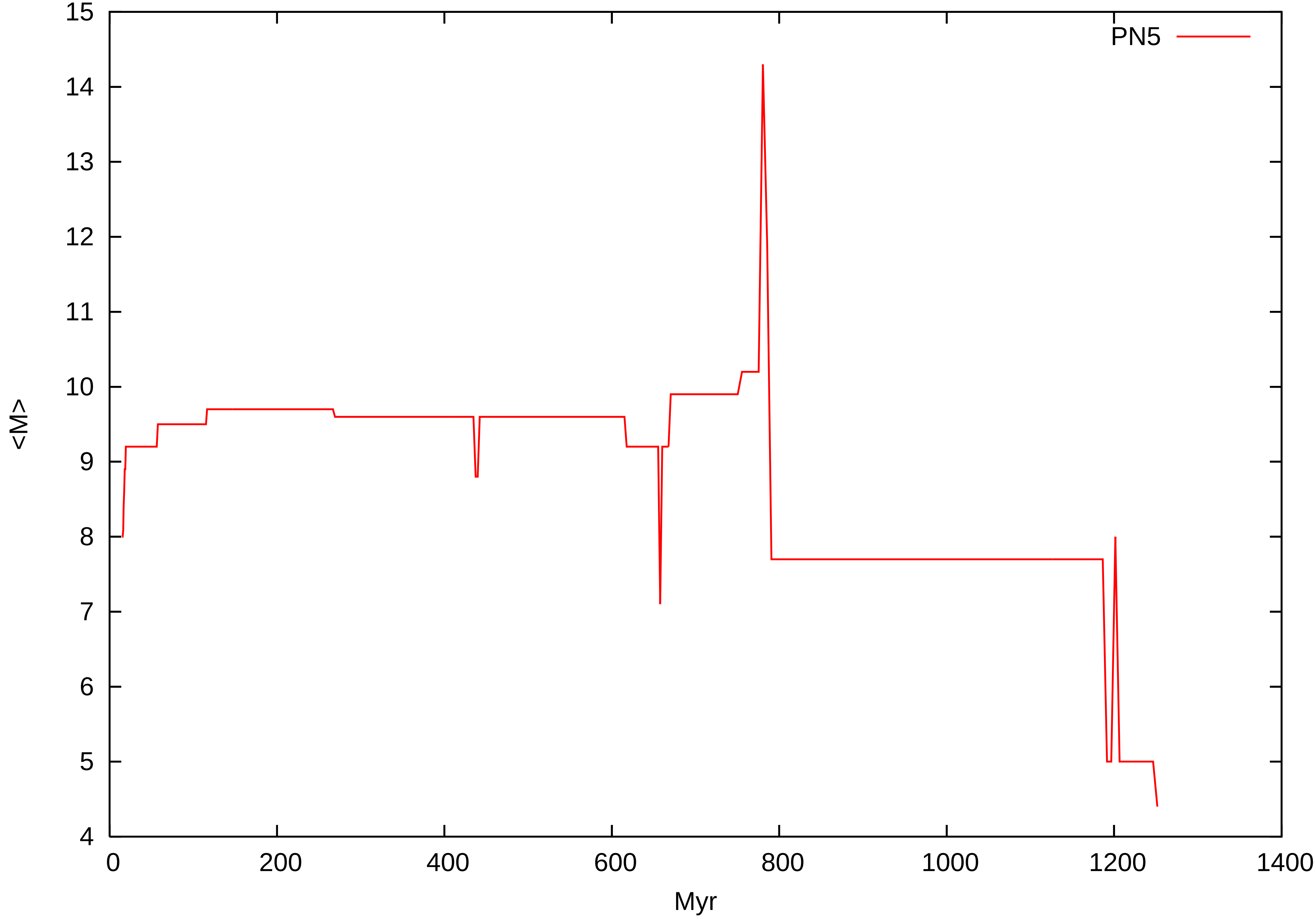
M1 Model B3
M2 Model B3
r
Myr
0
0.2
0.4
0.6
0.8
1
1.2
0
2
4
6
8
10
12
14
16



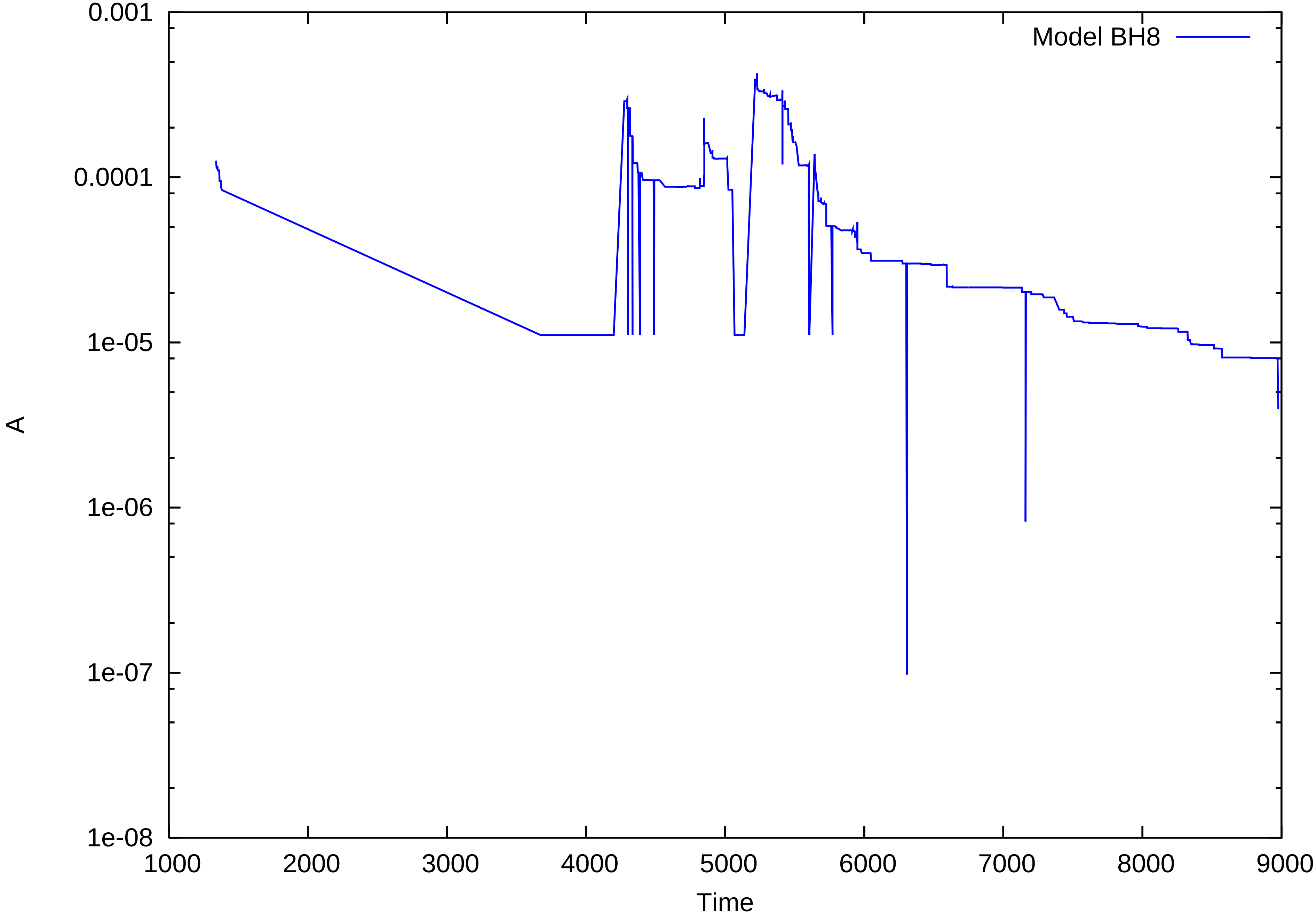
Model R7
VESC
Myr
0
10
20
30
40
50
60
70
80
90
100
60
80
100
120
140
160
180
200



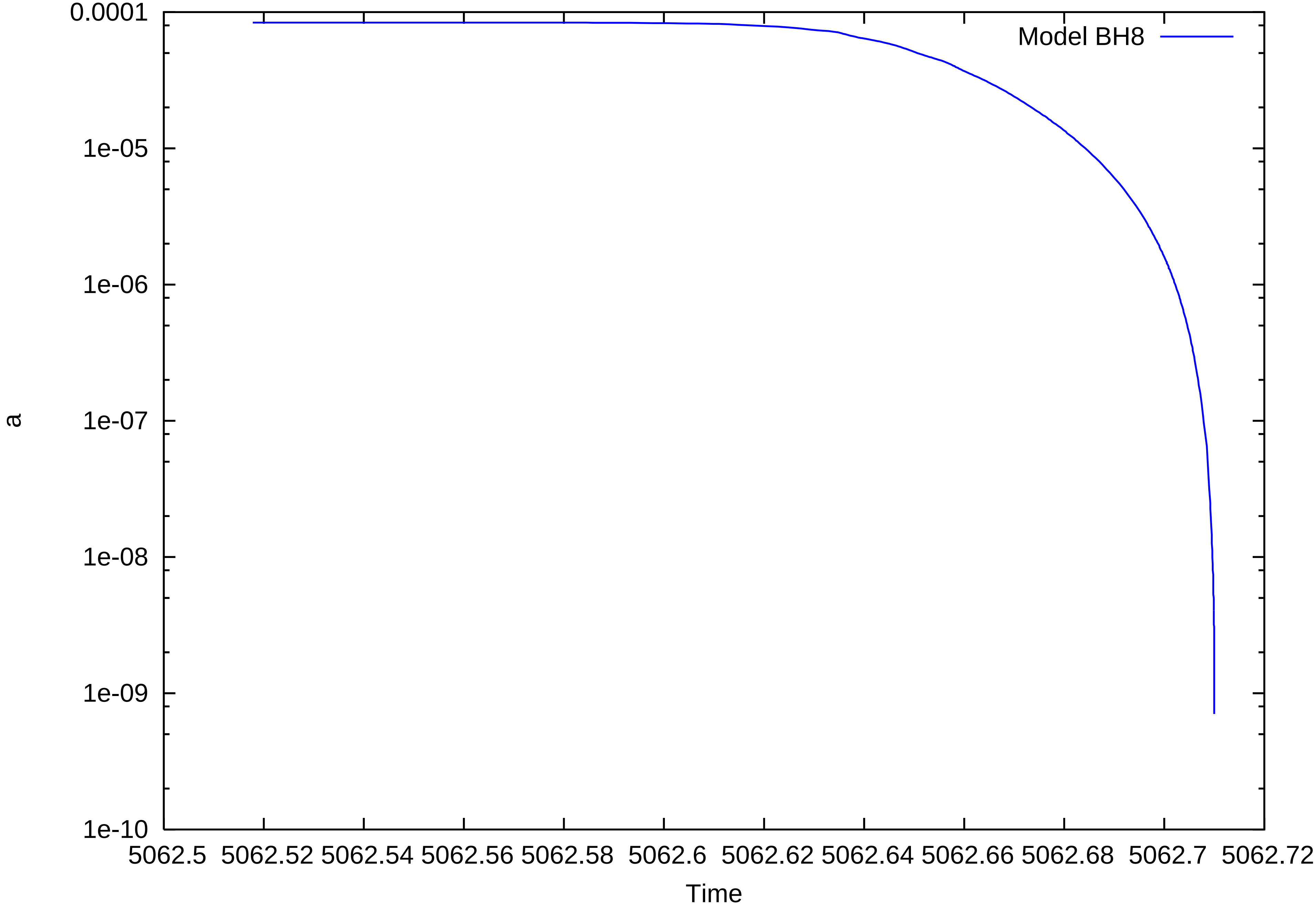
Model PN5
NBH
Myr
18
16
14
12
10
8
6
4
2
0
200
400
600
800
1000
1200
1400



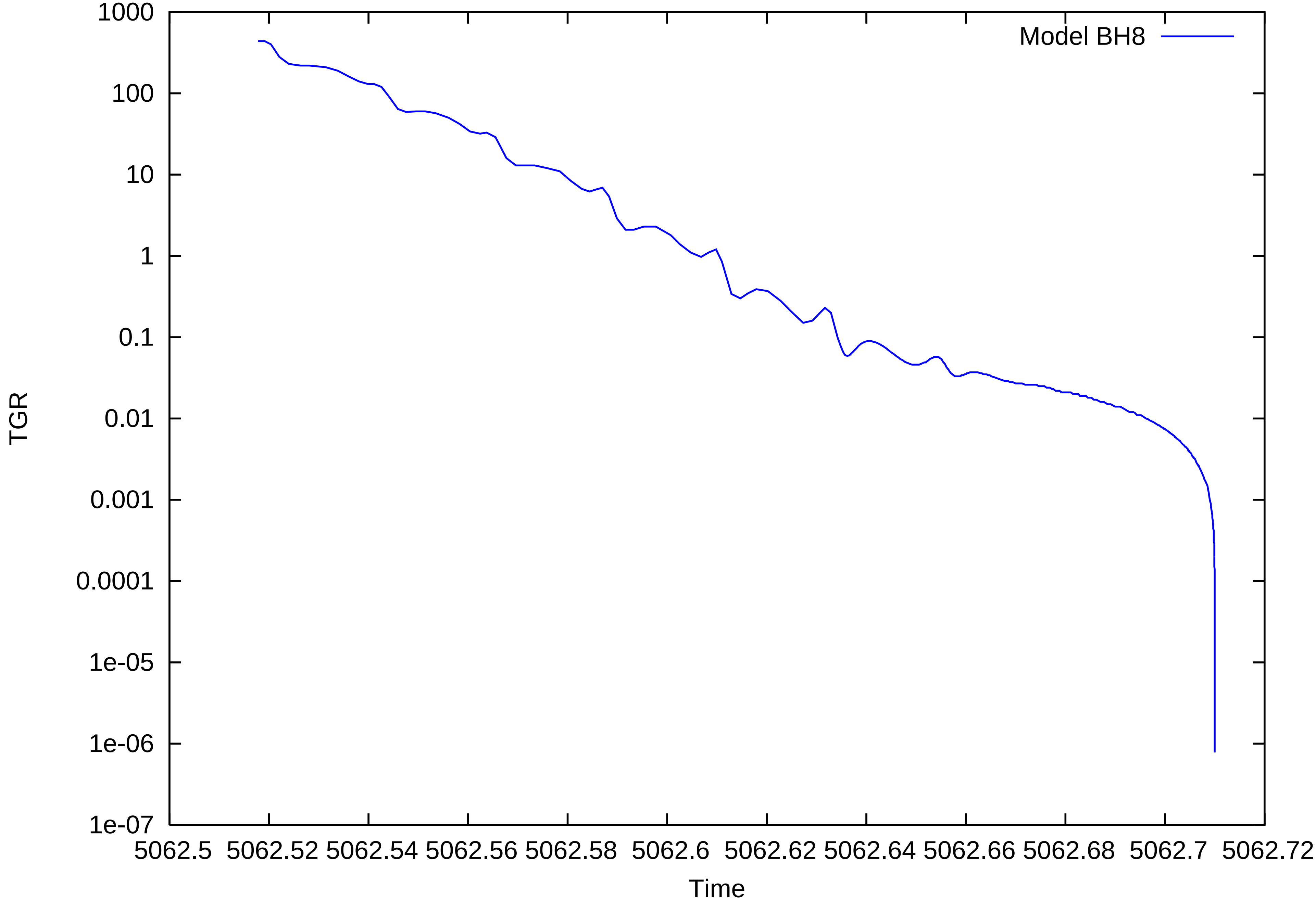
PN5
<M>
Myr
15
14
13
12
11
10
9
8
7
6
5
4
0
200
400
600
800
1000
1200
1400



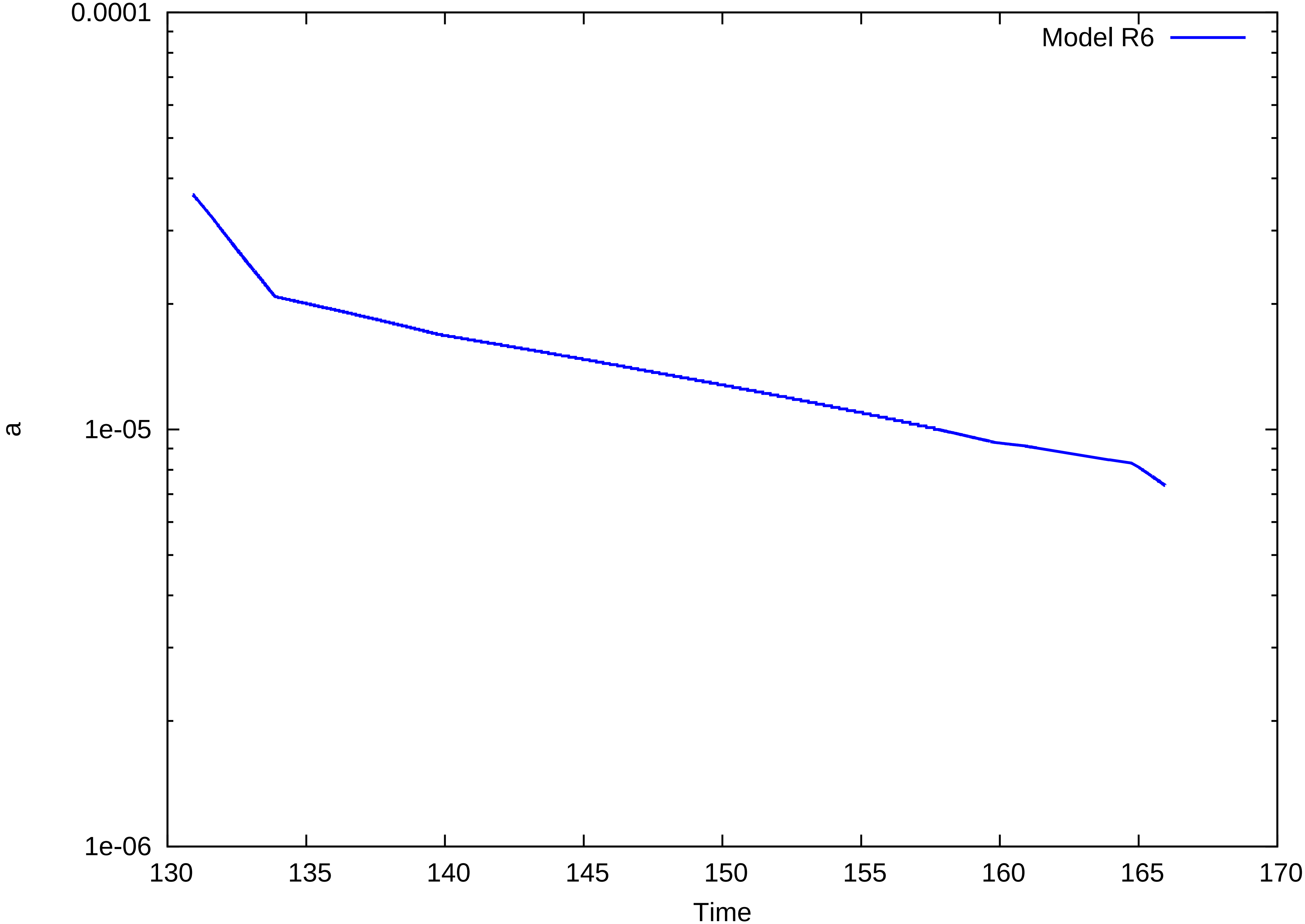
Model BH8
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1000
2000
3000
4000
5000
6000
7000
8000
9000
Time
A



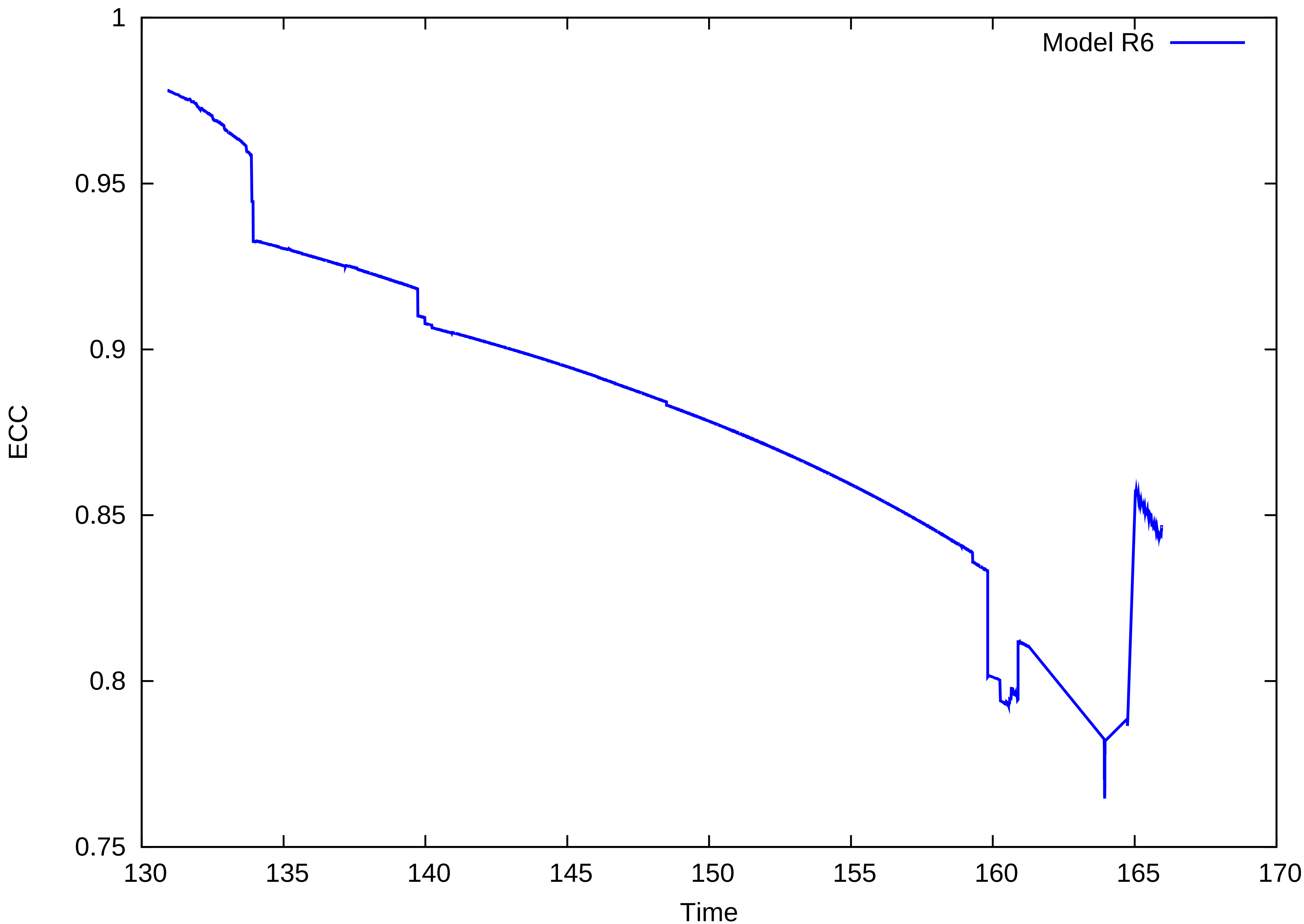
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
a
5062.5
5062.52
5062.54
5062.56
5062.58
5062.6
5062.62
5062.64
5062.66
5062.68
5062.7
5062.72
Time
Model BH8



Model BH8
TGR
Time
1000
100
10
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
5062.5
5062.52
5062.54
5062.56
5062.58
5062.6
5062.62
5062.64
5062.66
5062.68
5062.7
5062.72



Model R6
0.0001
1e-05
1e-06
a
130
135
140
145
150
155
160
165
170
Time


Model R6
ECC
Time
1
0.95
0.9
0.85
0.8
0.75
130
135
140
145
150
155
160
165
170

Outcome 1

Initial conditions	$N = 10\ 000$, Kroupa IMF in 0.2–12 $M_\odot$
	$NBH = 2$, $m_1 = m_2 = 15$, $R_{\rm Sch} = 4 \times 10^{-9}$
Binary with ARC	$t_0 = 16$, $t_{\rm f} = 52\,{\rm Myr}$
PN shrinkage	$t_{PN} = 30$, $a = 7 \times 10^{-6}$, $e = 0.85$
Strong interaction	$t_{\rm f} = 52$, $a_{\rm f} = 5 \times 10^{-6}$, $e = 0.40$
Final escape	$m_1 = 4$, $m_{\rm b} = 30$, $v_{\rm esc} = 100,\ 12$, $t_{\rm GR} = 60\,{\rm Myr}$